NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2022-2023

BR2207 - QUANTITATIVE ANALYSIS

November 2022 Time Allowed: 3 hours

INSTRUCTIONS

1 This paper contains **TEN (10)** questions and comprises **SIX (6)** pages.

2 Answer **ALL** questions.

3 A copy of *Statistical Tables and Selected Distributions* is provided separately.

4 This is a **closed-book** examination.

5 The number of marks allocated is shown at the end of each question.

6 Begin your answer to each question on a separate page of the answer book.

7 Answers will be graded for content and appropriate presentation.

Note: Exam Questions begin on Page 2

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Question 1

A restaurant provides customers with options to customize their dinner menus. One customer can have a "full-menu", which is to choose one of the available options for each course, i.e. Entrée, Main Course, and Dessert. There are 3 available options for Entrée, 5 options for Main Course, and 6 options for Dissert.

(a) How many different ways can one customer choose for a full-menu dinner?

(2 marks)

(b) A customer can have a supreme dinner, by adding the following to the full-menu dinner: choosing 1 additional option for Entrée and choosing 2 additional options for Dissert. Any two chosen options **cannot** be the same. How many different ways can one customer choose for a supreme dinner?

(2 marks)

(c) A customer can have a light dinner, by removing one or two courses from the full-menu dinner. How many different ways can one customer choose for a light dinner?

(2 marks)

(TOTAL: 6 marks)

Question 2

A publicly listed company needs to be rated by a rating agency each year. Next year its rating can be "upgraded" by with a probability of 0.3; "downgraded" with a probability of 0.2; or "unchanged" with a probability of 0.5. You are also given the following information:

- If next year's rating is upgraded, the 2nd year's rating can be "upgraded" with a probability of 0.5 or "unchanged" with a probability of 0.5;
- If next year's rating is downgraded, the 2nd year's rating can be "downgraded" with a probability of 0.5 or "unchanged" with a probability of 0.5;
- If next year's rating is unchanged, the 2nd year's rating distribution is the same as that of the first year.

In each year, upgrading or downgrading can only be by 1 rating difference.

(a) Determine the sample space for possible rating outcomes of this company for the next two years.

(2 marks)

(b) Calculate the probability that this company is downgraded twice in the next two years.

(2 marks)

(c) Calculate the probability that this company's rating in Year 2 is the same as its current rating.

(2 marks)

(TOTAL: 6 marks)

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Question 3

You are hired by a cancer research centre to investigate the correlation between UV index and skin cancer incidence from a group of people. You have the following table that depicts the joint probability mass function for Skin Cancer Incidence (X) and UV Index (Y).

		Skin Cancer Incidence (X)		
		100	300	500
UV Index (Y)	2	0.1	0.15	0.05
	5	0.1	0.1	0.1
	8	0.05	0.15	0.2

(a) Determine the expected skin cancer incidence.

(2 marks)

(b) Determine the expected skin cancer incidence given UV index is 8.

(2 marks)

(c) Determine the covariance between UV index and skin cancer incidence. Interpret the result in words.

(4 marks)

(TOTAL: 8 marks)

Question 4

A scientist studies the mortality of mouse in an experiment, where a group of 10 homogeneous mice were selected. The scientist assumes that each mouse's life status follows a Bernoulli distribution with probability of death within one week equal to q; and that all mice are independent. The scientist then focuses on the number of deaths observed within one week.

(a) State which distribution, including parameters, is followed by the number of deaths from the group of mice within the first week?

(2 marks)

(b) Show that $\hat{\theta} = \frac{X}{10}$ is an unbiased estimator of q, where X is the number of observed deaths within the first week.

(3 marks)

(c) At the end of Week 1, the scientist found that 2 mice died within the first week. The scientist uses the unbiased estimator of q in Question (b) based on observations in Week 1. She believes that the surviving mice's death probabilities in Week 2 will be the same as in Week 1. What is the probability that the scientist observes more than 2 deaths of mice in Week 2?

(5 marks)

(TOTAL: 10 marks)

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Question 5

An investment fund manager uses Lognormal (LN) distribution to model stock price, whose current price is \$100. Let $S_0 = 100$ denote the current stock price and S_t denote the stock price at the end of Year t. She assumes the following distribution of future stock prices for any integer $t \geq 0$:

$$\ln\left(\frac{S_{t+1}}{S_t}\right) \sim N(\mu, \sigma^2)$$

Stock price returns in different years are mutually independent. After doing market research and data analysis, the fund manager believes that $\mu = 5\%$, and $\sigma = 20\%$.

(a) Calculate the probability that the stock price at the end of Year 1 is lower than its current price.

(2 marks)

(b) Show that given the current stock price of 100, the stock price at the end of Year t ($t > 0$) follows the following Lognormal distribution:

$$S_t \sim LN(\ln(100) + \mu t,\ \sigma^2 t)$$

(4 marks)

(c) The fund investors are concerned with extreme losses of investing into this stock, so they ask the fund manager to calculate stock price in extreme scenarios. Calculate the 95% Value-at-Risk of this stock's price at the end of Year 5. Interpret the result in words.

(6 marks)

(TOTAL: 12 marks)

Question 6

The expected remaining lifetime of a 70-year-old retiree in a large retirement village is 25 years, with a standard deviation of 30 years. A statistician randomly selected 36 death records from the retirement village.

State additional assumptions for using Central Limit Theorem (CLT) and use CLT to find the approximate probability that the average death age of the selected 36 records exceeds 100 years.

(TOTAL: 7 marks)

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Question 7

You are tasked with a study on the number of car accidents in Country M, using historical data of monthly car accidents in the past 10 years. You assume that the number of car accidents in each month follows a Poisson distribution with a constant parameter.

(a) Explain how you are going to use the Method of Moments to estimate the parameter of the Poisson distribution. Define notation where necessary.

(3 marks)

(b) Write out the log-likelihood function for obtaining the maximum likelihood estimate (MLE) of the Poisson distribution. Show that the MLE for the parameter of the Poisson distribution is the sample mean. Define notation where necessary.

(6 marks)

(c) Data of monthly car accidents in the past 120 months (10 years) is summarized in the table below. x_i denotes the number of car accidents in month i, where $i = 1,2,\ldots,120$.

No. of observations (n)	120
Sum of no. of car accidents ($\sum_{i=1}^{n} x_i$)	1,200
Sum of squared no. of car accidents ($\sum_{i=1}^{n} {x_i}^2$)	14,380

Calculate sample mean and sample variance. Construct a 95% confidence interval for the mean of the population sampled (i.e. the parameter of the Poisson distribution).

(5 marks)

(d) Do you think Poisson distribution is a good choice for this task? Explain your answer.

(2 marks)

(TOTAL: 16 marks)

Question 8

The return of Stock B follows a Normal distribution with standard deviation of 20%. You want to test the Null Hypothesis that the mean return is 10%, against the Alternative Hypothesis that the mean return is 0%. You have n i.i.d random samples of Stock B's returns, with a sample mean of $\bar{X}$. You will accept the Null Hypothesis if $\bar{X} > K$; otherwise, you will reject it.

(a) Determine Type I error and Type II error, as functions of K and n. Comment on how the values of the two types of error will change if you increase the values of K ($K < 10\%$) and n, respectively.

(8 marks)

(b) Assume $K = 5\%$. You want to reduce both types of error to 5% or lower. Show that the minimum required sample size is 44.

(4 marks)

(TOTAL: 12 marks)

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Question 9

As a newly qualified FRM working at the World Bank, you are comparing the central bank interest rates of two countries. You have observed the following information: (1) 8 interest rates of Country A, with a sample mean of 2%, and sample standard deviation of 3%; (2) 8 interest rates of Country B, with a sample mean of 1%, and sample standard deviation of 2%.

Assuming that interest rates of the two countries are independent, follow Normal distributions and have equal variances, you want to test if the mean interest rate in Country A is greater than the mean interest rate of Country B.

(a) Since the sample sizes are small, what test are you going to use?

(2 marks)

(b) Set up the null and alternate hypothesis for testing if the mean interest rate in Country A is greater than that of Country B, hence proceed with the hypothesis test at the 5% level of significance.

(8 marks)

(TOTAL: 10 marks)

Question 10

You are an investment analyst and looking to invest into a market-neutral hedge fund, i.e. fund investment returns are not affected by market performance ($\beta = 0$). You found one potential hedge fund (called GeniusFund) and performed the following simple regression model:

$$y = \alpha + \beta x + \epsilon,$$

where x corresponds to market annual excess return (i.e. market return minus risk-free rate), and y corresponds to GeniusFund's annual excess return (i.e. fund return minus risk-free rate).

You also obtained the following information for the past 10 years:

$$\sum_{i=1}^{10} x_i = 0.5, \sum_{i=1}^{10} x_i^2 = 0.2, \quad \sum_{i=1}^{10} y_i = 0.2, \sum_{i=1}^{10} y_i^2 = 0.05, \quad \sum_{i=1}^{10} x_i y_i = 0.08$$

(a) Find the fitted values of the two parameters in the simple regression.

(4 marks)

(b) Calculate the coefficient of determinant. Interpret your result.

(3 marks)

(c) Construct the 95% confidence interval for the slope coefficient of the regression model. Based on your result, will you invest into GeniusFund?

(6 marks)

(TOTAL: 13 marks)

- END OF PAPER -

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BR2207 QUANTITATIVE ANALYSIS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

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