Tutorial 2

1 GARP

Bob makes purchases to maximize an increasing utility function and therefore his choices must satisfy GARP. We observe Bob buy $x = (x_1, x_2)$ when prices are $p = (p_1, p_2)$ where $x_1 > 0$ and $x_2 > 0$ and he spends $m = p \cdot x$.

1. I told you that GARP and WARP are equivalent when there are only two goods. Suppose there are three goods and the consumer buys $x^1 = (1, 0, 0)$, $x^2 = (0, 1, 0)$, and $x^3 = (0, 0, 1)$ at prices p^1, p^2, and p^3. Can you come up with price vectors p^1, p^2, and p^3 so that the consumer's behavior satisfies WARP but not GARP?

2. A utility function $U : \mathbb{R}^L_+ \to \mathbb{R}$ is *strictly concave* if

$$U(\alpha x + (1 - \alpha)x') > \alpha U(x) + (1 - \alpha)U(x') \tag{1}$$

   for all $x' \neq x$ and all α taking values strictly between 0 and 1. Show that if U is strictly concave then each budget set $B(p, m)$ can have at most one consumption bundle which maximizes the utility function. In other words, there is some $x \in B(p, m)$ so that $U(x) > U(x')$ for all $x' \in B(p, m)$ so that $x \neq x'$. Hint: suppose there are two bundles x and x' which maximize utility U over all bundles in $B(p, m)$ and show that this contradicts (1).

3. A consumer satisfies the Strong Axiom of Revealed Preferences (SARP) if $x \; R^* \; x'$ and $x' \; R \; x$ then $x = x'$. Suppose that a consumer maximizes a strictly concave utility function (that is, for each t we have $U(x^t) \geqslant U(x)$ for all $x \in B(p^t, p^t \cdot x^t)$) and that $x^1 \; R \; x^2 \; R \; x^3 \; R \; x^4 \; R \ldots R \; x^T \; R \; x^1$. Show that $x^1 = x^2 = x^3 = \ldots = x^T$.

4. When prices are $p = (p_1, p_2)$ Bob purchases $x = (x_1, x_2)$ where $x_1 > 0$. Let $m = p \cdot x$. Prices change to $p' = (p'_1, p_2)$ with $p'_1 > p_1$ (note the price of good 2 is unchanged). Bob buys some bundle x' which satisfies $p' \cdot x' = m$. Show that his behavior satisfies GARP.

5. As in the previous question suppose that prices change to $p' = (p_1', p_2)$ with $p_1' > p_1$ but suppose that expenditure changes to $m' = p' \cdot x$. Suppose the consumer satisfies GARP. Show that his demand x' at the new price p' must satisfy $p \cdot x' \geqslant p \cdot x$.

2 Demand example

Consider the utility function

$$U(x) = \sqrt{x_1} + \sqrt{x_2}$$

This U generates the demand function

$$x(p_1, p_2, m) = \frac{m}{p_1 + p_2}\left(\frac{p_2}{p_1}, \frac{p_1}{p_2}\right)$$

1. What consumption bundle does the consumer demand when $p = (1, 1)$ and $m = 1$.

2. Suppose the price of good 1 rises. Does the quantity demanded for good 2 rise / fall or cannot say? Hint: you can write the demand for good 2 as

$$x_2(p_1, p_2, m) = \left(\frac{p_1}{p_1 + p_2}\right)\frac{m}{p_2}$$

3. Are the goods normal, inferior, or neither?

4. The original price and expenditure is $p = (p_1, p_2)$ and m. Suppose the price of good 1 rises to $p_1' > p_1$ and so prices are $p' = (p_1', p_2)$. Calculate the Slutsky substitution and income effects. That is, calculate

$$\Delta x_1^s \qquad \text{and} \qquad \Delta x_1^m$$

   What can you say about the sign of these terms?

3 Perfect Complements

Consider the utility function

$$U(x_1, x_2) = \min(x_1, x_2)$$

So, $U(2,3) = 2$, $U(6,1) = 1$, $U(100,3) = 3$ and so on.

1. Draw some indifference curves for this utility function.

2. Find the demand function for this utility function. Hint: demand ought to satisfy $p \cdot x(p,m) = m$ and $x_1(p,m) = x_2(p,m)$.

3. Are the goods normal, inferior, or neither?

4. The original price and expenditure is $p = (1,1)$ and $m = 2$. Suppose the price of good 1 rises to 2 and so prices are $p' = (2,1)$. Calculate the Slutsky substitution and income effects. That is, calculate

$$\Delta x_1^s \qquad \text{and} \qquad \Delta x_1^m$$