Tutorial 3

1 Slutsky and Hicksian Compensation

1. A utility maximizing consumer has demand function $x(p_1, p_2, m)$. The price is $p = (p_1, p_2)$. The price of good 1 rises to $p_1' > p_1$. Show that the Slutsky compensation m_s lies above the Hicksian compensation m_h. That is, show that $m_s \leqslant m_h$.

2 Utility maximization

Suppose $U(x_1, x_2) = x_1 x_2$.

1. Calculate the marginal utility for good 1 and 2: MU_1 and MU_2.
2. Calculate the marginal rate of substitution for goods 1 and 2: MRS_{12}.
3. How much of good 1 will the consumer demand when prices are $(p_1, p_2) = (2, 1)$ and the expenditure is $m = 1$. Hint: use the budget constraint $p_1 x_1 + p_2 x_2 = m$
4. Calculate the demand function $x(p_1, p_2, m)$.
5. Suppose endowment is $\omega = (3, 1)$. How much will the consumer demand when $(p_1, p_2) = (1, 3)$.

Suppose $U(x_1, x_2) = x_1 + 2\sqrt{x_2}$. The marginal utility is $\text{MU}_1 = 1$ and $\text{MU}_2 = x_2^{-1/2}$.

1. Calculate the marginal rate of substitution for goods 1 and 2: MRS_{12}.
2. How much of good 1 will the consumer demand when prices are $(p_1, p_2) = (2, 1)$ and the expenditure is $m = 1$.
3. Calculate the demand function $x(p_1, p_2, m)$.
4. Suppose endowment is $\omega = (3, 1)$. How much will the consumer demand when $(p_1, p_2) = (2, 1)$.

3 Demand example

Consider the utility function

$$U(x) = \sqrt{x_1} + \sqrt{x_2}$$

This U generates the demand function

$$x(p_1, p_2, m) = \frac{m}{p_1 + p_2}\left(\frac{p_2}{p_1}, \frac{p_1}{p_2}\right)$$

Suppose the consumer with utility function U has endowment $\omega = (3, 1)$.

1. Show that $p \cdot z(p) = 0$ where z is the net demand function (i.e. $z(p) = x(p, p \cdot \omega) - \omega$).
2. What is the consumer's net demand $z(p)$ at $p = (1, 1)$? Is the consumer a net buyer or seller of good 1?
3. Prices change to $p' = (2, 1)$. What is the net demand $z(p')$ at the new price level?
4. Calculate the income and substitution effects for good 1 for this price change.

4 Perfect Complements

Consider the utility function

$$U(x_1, x_2) = \min(x_1, x_2)$$

So, $U(2, 3) = 2$, $U(6, 1) = 1$, $U(100, 3) = 3$ and so on. The demand function is

$$x(p_1, p_2, m) = \left(\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2}\right)$$

and the consumer has endowment $\omega = (3, 1)$.

1. Show that $p \cdot z(p) = 0$ where z is the net demand function (i.e. $z(p) = x(p, p \cdot \omega) - \omega$).
2. What is the consumer's net demand $z(p)$ at $p = (1, 1)$? Is the consumer a net buyer or seller of good 1?

3. Prices change to $p' = (2, 1)$. What is the net demand $z(p')$ at the new price level?

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