Tutorial 3

1 Slutsky and Hicksian Compensation

1. A utility maximizing consumer has demand function $x(p_1, p_2, m)$. The price is $p = (p_1, p_2)$. The price of good 1 rises to $p_1' > p_1$. Show that the Slutsky compensation m_s lies above the Hicksian compensation m_h. That is, show that $m_s \geqslant m_h$.

Answer. By definition $U(x(p, m)) = U(x(p', m_h))$ and because original bundle $x(p, m)$ can be bought when facing prices p' at income m_s we know $U(x(p', m_s)) \geqslant U(x(p', m)) = U(x(p', m_h))$. Must be that budget set (p', m_s) contains budget set (p', m_h) and so $m_s \geqslant m_h$.

2 Utility maximization

Suppose $U(x_1, x_2) = x_1 x_2$.

1. Calculate the marginal utility for good 1 and 2: MU_1 and MU_2.

   Answer.

$$\frac{U(x_1 + \alpha, x_2) - U(x_1, x_2)}{\alpha} = \frac{(x_1 + \alpha)x_2 - x_1 x_2}{\alpha} = x_2 = MU_1$$

2. Calculate the marginal rate of substitution for goods 1 and 2: MRS_{12}.

   Answer. $\text{MRS}_{12} = MU_1/MU_2 = x_2/x_1$.

3. How much of good 1 will the consumer demand when prices are $(p_1, p_2) = (2, 1)$ and the expenditure is $m = 1$. Hint: use the budget constraint $p_1 x_1 + p_2 x_2 = m$

   Answer. Use demand function from next answer if you like. You'll get 1/4.

4. Calculate the demand function $x(p_1, p_2, m)$.

   $x_2/x_1 = p_1/p_2$ which implies $p_2 x_2 = p_1 x_1$ putting into the budget constraint gives $2p_1 x_1 = m$ and so $x_1(p, m) = m/(2p_1)$. Similarly, $x_2(p, m) = m/(2p_2)$.

5. Suppose endowment is $\omega = (3, 1)$. How much will the consumer demand when $(p_1, p_2) = (1, 3)$.

   Answer. You have $(1, 3) \cdot (3, 1) = 6$ to spend. So, demand $x(1, 3, 6) = (3, 1)$.

Suppose $U(x_1, x_2) = x_1 + 2\sqrt{x_2}$. The marginal utility is $\mathrm{MU}_1 = 1$ and $\mathrm{MU}_2 = x_2^{-1/2}$.

1. Calculate the marginal rate of substitution for goods 1 and 2: MRS_{12}.

   Answer. $MU_1/MU_2 = x_2^{1/2}$.

2. How much of good 1 will the consumer demand when prices are $(p_1, p_2) = (2, 1)$ and the expenditure is $m = 10$.

   Answer. Have $x_2^{1/2} = p_1/p_2$ and so $x_2 = (p_1/p_2)^2$. Demand for good 1 is $\frac{m}{p_1} - p_1/p_2$. So, $10/2 - 2 = 3$.

3. Calculate the demand function $x(p_1, p_2, m)$.

   Answer. See previous answer.

4. Suppose endowment is $\omega = (3, 3)$. How much will the consumer demand when $(p_1, p_2) = (2, 1)$.

   Answer. Consumer has $(3, 3) \cdot (2, 1) = 9$ to spend. Demand $(2.5, 4)$.

3 Demand example

Consider the utility function

$$U(x) = \sqrt{x_1} + \sqrt{x_2}$$

This U generates the demand function

$$x(p_1, p_2, m) = \frac{m}{p_1 + p_2}\left(\frac{p_2}{p_1}, \frac{p_1}{p_2}\right)$$

Suppose the consumer with utility function U has endowment $\omega = (3, 1)$.

1. Show that $p \cdot z(p) = 0$ where z is the net demand function (i.e. $z(p) = x(p, p \cdot \omega) - \omega$).

Answer. $p \cdot z(p) = p \cdot (x(p, p \cdot \omega) - \omega) = p \cdot x(p, p \cdot \omega) - p \cdot \omega$. So we have $p \cdot z(p) = 0$ if $p \cdot x(p, p \cdot \omega) = p \cdot \omega$. We have

$$p \cdot x(p, p \cdot \omega) = \frac{p \cdot \omega}{p_1 + p_2}(p_1, p_2) \cdot \left(\frac{p_2}{p_1}, \frac{p_1}{p_2}\right) = \frac{p \cdot \omega}{p_1 + p_2}(p_2 + p_1) = p \cdot \omega$$

2. What is the consumer's net demand $z(p)$ at $p = (1, 1)$? Is the consumer a net buyer or seller of good 1?

   Answer. $x(1, 1, 4) = \frac{4}{2}(1, 1) = (2, 2)$ and so $z(p) = x(1, 1, 4) - (3, 1) = (-1, 1)$.

3. Prices change to $p' = (2, 1)$. What is the net demand $z(p')$ at the new price level?

   Answer. $p \cdot \omega = 7$ and so $x(2, 1, 7) = \frac{7}{3}(\frac{1}{2}, 2) = (\frac{7}{6}, \frac{14}{3})$. Net demand is $x(2, 1, 7) - (3, 1) = (-\frac{11}{6}, \frac{11}{3})$

4. Calculate the income and substitution effects for good 1 for this price change.

   Answer. $m_s = p' \cdot x(p, 4) = 6$. So, $x(p', 6) = \frac{6}{3}(\frac{1}{2}, 2) = (1, 4)$. So,

$$\Delta x_1^s = 1 - 2 = -1, \qquad \text{and} \qquad \Delta x_1^n = \tfrac{7}{6} - 1 = \tfrac{1}{6}$$

4 Perfect Complements

Consider the utility function

$$U(x_1, x_2) = \min(x_1, x_2)$$

So, $U(2, 3) = 2$, $U(6, 1) = 1$, $U(100, 3) = 3$ and so on. The demand function is

$$x(p_1, p_2, m) = \left(\frac{m}{p_1 + p_2}, \frac{m}{p_1 + p_2}\right)$$

and the consumer has endowment $\omega = (3, 1)$.

1. Show that $p \cdot z(p) = 0$ where z is the net demand function (i.e. $z(p) = x(p, p \cdot \omega) - \omega$).

   Answer. Can show that $x(p, p \cdot \omega) = p \cdot \omega$. $p \cdot x(p, m) = p_1 p \cdot \omega/(p_1 + p_2) + p_2 p \cdot \omega/(p_1 + p_2) = p \cdot \omega(p_1 + p_2)/(p_1 + p_2) = p \cdot \omega$

2. What is the consumer's net demand $z(p)$ at $p = (1, 1)$? Is the consumer a net buyer or seller of good 1?

   Answer. $m = p \cdot \omega = 4$ and so $x(1, 1, 4) = (2, 2)$. Net demand $(-1, 1)$. Net seller of good 1.

3. Prices change to $p' = (2, 1)$. What is the net demand $z(p')$ at the new price level?

   Answer. $m' = p' \cdot \omega = 7$. So, $x(2, 1, 7) = (\frac{7}{3}, \frac{7}{3})$. Net demand $(-\frac{2}{3}, \frac{4}{3})$.

4. Calculate the income and substitution effects for good 1 for this price change.

   Answer. $m_s = p' \cdot (2, 2) = 6$. So, $x(p', 6) = (2, 2)$. So, $\Delta x_1^s = 0$ and $\Delta x_1^n = \frac{7}{3} - 2 = \frac{1}{3}$