HE2001 Microeconomics II

Tutorial 8

Academic Year 2025/2026, Semester 2

Quantitative Research Society @NTU

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Overview

This tutorial covers:

- per-unit subsidies, statutory incidence, and the role of supply and demand elasticities;
- ad-valorem taxation and welfare decomposition into consumer surplus, producer surplus, tax revenue, and deadweight welfare loss;
- horizontal summation of individual demand and supply curves in heterogeneous-consumer and multi-firm markets;
- equilibrium, consumer surplus, producer surplus, and deadweight loss under per-unit taxes;
- comparative tax incidence as the number of firms changes;
- past-year style applications involving quasilinear demand, kinked aggregate demand, and zero-trade equilibrium.

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Contents

Question 1: Per-Unit Subsidies **3**
Problem . . . 3
Solution . . . 4

Question 2: Ad-Valorem Tax in the Car Market **6**
Problem . . . 6
Solution . . . 6
Method 1: Solve directly from the wedge condition . . . 6
Method 2: Rewrite demand in terms of the sellers' price . . . 8

Question 3: Bubble Tea Market Aggregation **10**
Problem . . . 10
Solution . . . 11
Method 1: Horizontal summation in direct-demand form . . . 11
Method 2: Inverse-curve representation . . . 14

Question 4: Mala-Hotpot Excise Tax **16**
Problem . . . 16
Solution . . . 16
Method 1: Aggregate supply then equate demand and supply . . . 16
Method 2: Inverse supply representation . . . 19

Sample Question 1: Good X with Heterogeneous Consumers and Firms **21**
Problem . . . 21
Solution . . . 22
Method 1: Direct optimisation, aggregation, and equilibrium solving . . . 22
Method 2: Inverse-demand / inverse-supply and welfare geometry . . . 28
Method 3: Comparative-statics / elasticity and regime-switch method . . . 31
Marking scheme (40 marks) . . . 33

Sample Question 2: Teddy Bears **35**
Problem . . . 35
Solution . . . 36
Method 1: Direct aggregation and tax-wedge algebra . . . 36
Method 2: Inverse-demand / inverse-supply and welfare geometry . . . 39
Method 3: Concise olympiad-style tax-parameter method . . . 42
Marking scheme (30 marks) . . . 44

Question 1: Per-Unit Subsidies

Problem

Let us consider the case of a per unit subsidy.

Assuming that we have linear supply curves and demand curves. Illustrate using a diagram:

For a) and b), indicate in the diagram(s) consumer surplus, producer surplus and amount of deadweight welfare loss.

(a) The effects of a per unit subsidy of s on producers.

(b) The effects of a per unit subsidy of s on consumers.

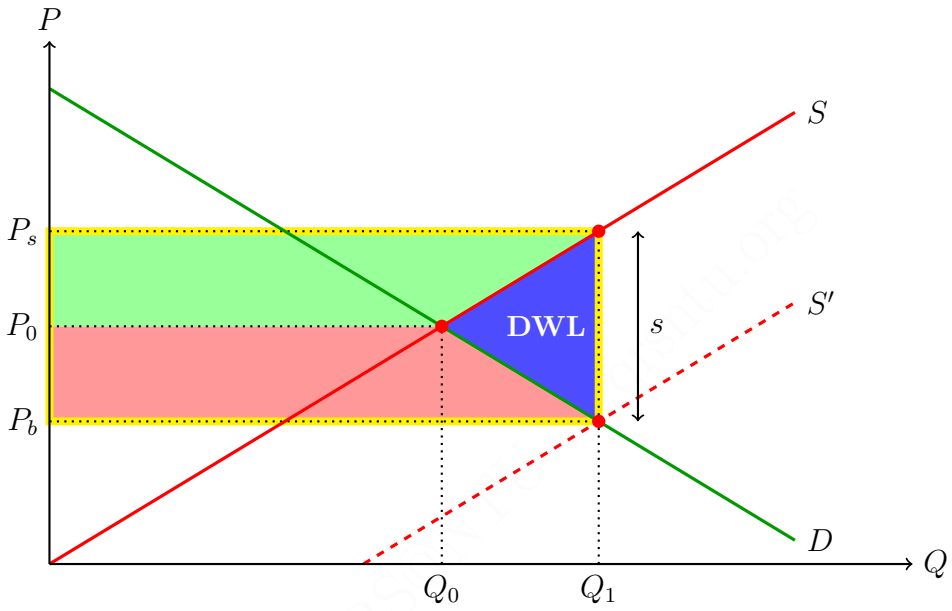
(c) How do you think the elasticity of supply and demand will affect the incidence of benefits of a per unit subsidy on producers?

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Solution

(a) If the subsidy is paid to producers, then for each unit sold producers receive the buyers' price plus the subsidy. If P_b denotes the price paid by buyers and P_s denotes the price received by sellers net of the subsidy, then

$$P_s = P_b + s.$$

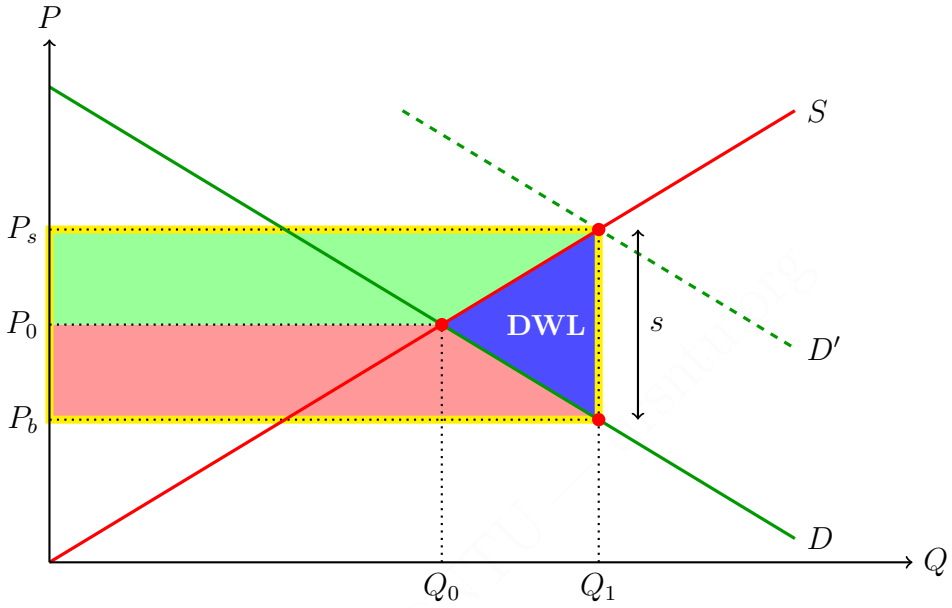


The economic effects are:

Q rises, P_b falls, P_s rises, and DWL appears due to overproduction.

(b) If the subsidy is paid to consumers, then consumers pay the market price to sellers but recover s per unit from the government. If P_s denotes the price received by sellers and P_b denotes the net price borne by buyers, then

$$P_s = P_b + s.$$



Thus

Q rises, P_b falls, P_s rises, and deadweight loss appears due to overconsumption or overproduction.

(c) The side of the market that is relatively less elastic captures more of the benefit from a subsidy. Therefore producers receive a larger share of the subsidy when:

- supply is relatively more inelastic; and/or
- demand is relatively more elastic.

The intuition is standard. If producers are not very responsive to price, quantity supplied changes only a little, so more of the subsidy shows up as a higher price received by producers rather than as a lower price paid by consumers. If consumers are very price-sensitive, buyers' prices do not need to fall much to generate the additional quantity, so more of the subsidy remains with producers.

Hence

Producers capture more of the subsidy when supply is more inelastic and demand is more elastic.

Question 2: Ad-Valorem Tax in the Car Market

Problem

In the market for cars, the market demand curve is given by $D(p) = 1200 - 100P$ while market supply curve is given by $S(p) = 100p$.

(a) Suppose that there is an ad-valorem (percentage) sales tax of 5 percent paid by consumers. I.e. $P_B = 1.05P_S$, where P_B is the price paid by buyers and P_S is the price received by sellers. Solve for the equilibrium P_S and P_B and quantity in the market.

(b) Illustrate the effects of the ad-valorem tax above in a diagram, indicating the consumer surplus, producer surplus and deadweight welfare loss.

Solution

Method 1: Solve directly from the wedge condition

(a) With the tax wedge,

$$P_B = 1.05P_S.$$

Equilibrium requires demand at the buyers' price equals supply at the sellers' price:

$$1200 - 100P_B = 100P_S.$$

Substitute $P_B = 1.05P_S$:

$$1200 - 100(1.05P_S) = 100P_S.$$

So

$$1200 - 105P_S = 100P_S,$$

hence

$$1200 = 205P_S \quad \Longrightarrow \quad P_S = \frac{1200}{205} = \frac{240}{41}.$$

Therefore

$$P_B = 1.05P_S = \frac{21}{20} \cdot \frac{240}{41} = \frac{252}{41}.$$

The equilibrium quantity is

$$Q = 100P_S = 100 \cdot \frac{240}{41} = \frac{24000}{41}.$$

Thus

$$\boxed{P_S = \frac{240}{41}, \qquad P_B = \frac{252}{41}, \qquad Q = \frac{24000}{41}.}$$

(b) First compute the no-tax benchmark. Without tax,

$$1200 - 100P = 100P \quad \Longrightarrow \quad P_0 = 6, \qquad Q_0 = 600.$$

In buyer-price space, the inverse demand and supply curves are

$$P = 12 - \frac{Q}{100}, \qquad P = \frac{Q}{100}.$$

With the ad-valorem tax, buyers pay

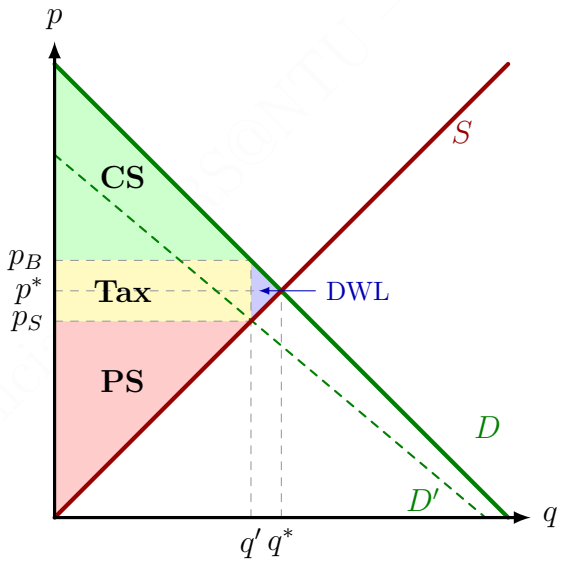
$$P_B = 1.05 P_S = 1.05 \cdot \frac{Q}{100},$$

so the tax-distorted supply curve in buyer-price space is

$$P_B = \frac{1.05Q}{100}.$$

The post-tax equilibrium is

$$(Q_t, P_B, P_S) = \left(\frac{24000}{41}, \frac{252}{41}, \frac{240}{41}\right).$$



The corresponding exact welfare areas are:

$$CS_t = \frac{1}{2} \cdot \frac{24000}{41} \left(12 - \frac{252}{41}\right) = \frac{2880000}{1681},$$

$$PS_t = \frac{1}{2} \cdot \frac{24000}{41} \cdot \frac{240}{41} = \frac{2880000}{1681},$$

$$\text{Tax revenue} = (P_B - P_S)Q_t = \left(\frac{12}{41}\right)\left(\frac{24000}{41}\right) = \frac{288000}{1681},$$

$$DWL = \frac{1}{2}(Q_0 - Q_t)(P_B - P_S) = \frac{1}{2}\left(600 - \frac{24000}{41}\right)\left(\frac{12}{41}\right) = \frac{3600}{1681}.$$

Hence

$$\boxed{CS_t = \frac{2880000}{1681}, \quad PS_t = \frac{2880000}{1681}, \quad DWL = \frac{3600}{1681}.}$$

Method 2: Rewrite demand in terms of the sellers' price

(a) Demand as a function of the sellers' price is

$$Q_D = 1200 - 100P_B = 1200 - 100(1.05P_S) = 1200 - 105P_S.$$

Supply is

$$Q_S = 100P_S.$$

Equating them,

$$1200 - 105P_S = 100P_S \quad \Longrightarrow \quad 205P_S = 1200 \quad \Longrightarrow \quad P_S = \frac{240}{41}.$$

Then

$$P_B = \frac{21}{20}P_S = \frac{252}{41}, \qquad Q = 100P_S = \frac{24000}{41}.$$

Hence

$$\boxed{P_S = \frac{240}{41}, \qquad P_B = \frac{252}{41}, \qquad Q = \frac{24000}{41}.}$$

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(b) Total-surplus comparison Before tax,

$$P_0 = 6, \qquad Q_0 = 600.$$

Thus

$$CS_0 = \frac{1}{2} \cdot 600 \cdot (12 - 6) = 1800, \qquad PS_0 = \frac{1}{2} \cdot 600 \cdot 6 = 1800.$$

So total surplus before tax is

$$TS_0 = 3600.$$

After tax,

$$TS_t = CS_t + PS_t + \text{Tax revenue} = \frac{2880000}{1681} + \frac{2880000}{1681} + \frac{288000}{1681} = \frac{6048000}{1681}.$$

Therefore

$$DWL = TS_0 - TS_t = 3600 - \frac{6048000}{1681} = \frac{3600}{1681}.$$

This matches the triangle calculation in Method 1.

Thus

$$\boxed{DWL = \frac{3600}{1681}.}$$

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Question 3: Bubble Tea Market Aggregation

Problem

In the bubble tea market, there are 2 consumers with different preferences and 10 homogeneous firms. Consumer 1 has inverse demand function $p = 20 - 4q_1$ while Consumer 2 has inverse demand function $p = 15 - 2q_2$. Each of the firms has inverse supply function $p = 5q_i$.

(a) Derive the (aggregate) market supply curve and market demand curve.

(b) What is the equilibrium quantity and price in the market? Calculate the amount of producer and consumer surplus in the market.

(c) Suppose there is a per-unit tax on firms of \$0.5. What is the deadweight welfare loss from the tax?

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Solution

Method 1: Horizontal summation in direct-demand form

(a) Consumer 1's inverse demand is

$$p = 20 - 4q_1 \quad \Longrightarrow \quad q_1 = \frac{20-p}{4}$$

for $0 \le p \le 20$, and $q_1 = 0$ for $p > 20$.

Consumer 2's inverse demand is

$$p = 15 - 2q_2 \quad \Longrightarrow \quad q_2 = \frac{15-p}{2}$$

for $0 \le p \le 15$, and $q_2 = 0$ for $p > 15$.

Hence the market demand is obtained by horizontal summation:

$$Q_D(p) = q_1(p) + q_2(p).$$

Therefore

$$Q_D(p) = \begin{cases} \dfrac{20-p}{4} + \dfrac{15-p}{2}, & 0 \le p \le 15, \\ \dfrac{20-p}{4}, & 15 < p \le 20, \\ 0, & p > 20. \end{cases}$$

Simplifying the first branch,

$$Q_D(p) = \begin{cases} \dfrac{50-3p}{4}, & 0 \le p \le 15, \\ \dfrac{20-p}{4}, & 15 < p \le 20, \\ 0, & p > 20. \end{cases}$$

Each firm's inverse supply is

$$p = 5q_i \quad \Longrightarrow \quad q_i = \frac{p}{5}.$$

With 10 identical firms,

$$Q_S(p) = 10 \cdot \frac{p}{5} = 2p.$$

Hence

$$\boxed{Q_D(p) = \begin{cases} \dfrac{50-3p}{4}, & 0 \le p \le 15, \\ \dfrac{20-p}{4}, & 15 < p \le 20, \\ 0, & p > 20, \end{cases} \qquad Q_S(p) = 2p.}$$

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(b) Solve equilibrium in direct-demand form and sum surpluses by agent Because the equilibrium price will turn out to be below 15, both consumers are active and the relevant demand branch is

$$Q_D(p) = \frac{50 - 3p}{4}.$$

Equate this with market supply:

$$\frac{50 - 3p}{4} = 2p.$$

Thus

$$50 - 3p = 8p \quad \Longrightarrow \quad 11p = 50 \quad \Longrightarrow \quad p^* = \frac{50}{11}.$$

Then

$$Q^* = 2p^* = 2 \cdot \frac{50}{11} = \frac{100}{11}.$$

So the equilibrium is

$$\boxed{p^* = \frac{50}{11}, \qquad Q^* = \frac{100}{11}.}$$

Now compute each consumer's demand at the equilibrium price.

For Consumer 1,

$$q_1^* = \frac{20 - p^*}{4} = \frac{20 - \frac{50}{11}}{4} = \frac{85}{22}.$$

For Consumer 2,

$$q_2^* = \frac{15 - p^*}{2} = \frac{15 - \frac{50}{11}}{2} = \frac{115}{22}.$$

Their consumer surpluses are:

$$CS_1 = \frac{1}{2} q_1^* (20 - p^*) = \frac{1}{2} \cdot \frac{85}{22} \cdot \frac{170}{11} = \frac{7225}{242},$$

$$CS_2 = \frac{1}{2} q_2^* (15 - p^*) = \frac{1}{2} \cdot \frac{115}{22} \cdot \frac{115}{11} = \frac{13225}{484}.$$

So total consumer surplus is

$$CS = CS_1 + CS_2 = \frac{7225}{242} + \frac{13225}{484} = \frac{27675}{484}.$$

Each firm's output is

$$q_i^* = \frac{p^*}{5} = \frac{10}{11}.$$

Each firm has a linear inverse supply through the origin, so producer surplus per firm is

$$PS_i = \frac{1}{2} q_i^* p^* = \frac{1}{2} \cdot \frac{10}{11} \cdot \frac{50}{11} = \frac{250}{121}.$$

With 10 firms,

$$PS = 10 PS_i = \frac{2500}{121}.$$

Therefore

$$\boxed{CS = \frac{27675}{484}, \qquad PS = \frac{2500}{121}.}$$

(c) Wedge-and-triangle calculation A per-unit tax of $t = 0.5$ on firms creates the wedge

$$p_b = p_s + t = p_s + \frac{1}{2}.$$

Each firm's supply becomes

$$q_i = \frac{p_s}{5} = \frac{p_b - \frac{1}{2}}{5},$$

so aggregate supply in terms of the buyers' price is

$$Q_S^t(p_b) = 10 \cdot \frac{p_b - \frac{1}{2}}{5} = 2p_b - 1.$$

The relevant demand branch remains

$$Q_D(p_b) = \frac{50 - 3p_b}{4}.$$

Equating demand and taxed supply,

$$\frac{50 - 3p_b}{4} = 2p_b - 1.$$

Therefore

$$50 - 3p_b = 8p_b - 4 \quad \Longrightarrow \quad 54 = 11p_b \quad \Longrightarrow \quad p_b = \frac{54}{11}.$$

Hence

$$Q_t = 2p_b - 1 = \frac{108}{11} - 1 = \frac{97}{11}.$$

Without tax,

$$Q_0 = \frac{100}{11}.$$

So the fall in quantity is

$$Q_0 - Q_t = \frac{100}{11} - \frac{97}{11} = \frac{3}{11}.$$

The deadweight loss is the triangle

$$DWL = \frac{1}{2} \cdot t \cdot (Q_0 - Q_t) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{11} = \frac{3}{44}.$$

Thus

$$\boxed{DWL = \frac{3}{44}.}$$

Unofficial solutions

Method 2: Inverse-curve representation

(a) For the demand side, note the kink occurs where Consumer 2 just drops out, namely at $p = 15$. At that price Consumer 1 still demands

$$q_1 = \frac{20 - 15}{4} = \frac{5}{4},$$

so the market-demand kink is at

$$(Q, p) = \left(\frac{5}{4}, 15\right).$$

Thus the inverse market demand is

$$p = \begin{cases} 20 - 4Q, & 0 \le Q \le \frac{5}{4}, \\ \dfrac{50 - 4Q}{3}, & \frac{5}{4} \le Q \le \frac{25}{2}. \end{cases}$$

For supply, since $Q_S = 2p$, the inverse market supply is

$$p = \frac{Q}{2}.$$

Therefore

$$\boxed{p_D(Q) = \begin{cases} 20 - 4Q, & 0 \le Q \le \frac{5}{4}, \\ \dfrac{50 - 4Q}{3}, & \frac{5}{4} \le Q \le \frac{25}{2}, \end{cases} \qquad p_S(Q) = \frac{Q}{2}.}$$

(b) Use inverse market curves and integrate From Question 3.1, the inverse market demand is

$$p_D(Q) = \begin{cases} 20 - 4Q, & 0 \le Q \le \frac{5}{4}, \\ \dfrac{50 - 4Q}{3}, & \frac{5}{4} \le Q \le \frac{25}{2}, \end{cases} \qquad p_S(Q) = \frac{Q}{2}.$$

Since $Q^* = \frac{100}{11} > \frac{5}{4}$, the relevant demand branch at equilibrium is

$$\frac{50 - 4Q}{3} = \frac{Q}{2}.$$

Multiplying through by 6,

$$100 - 8Q = 3Q \quad \Longrightarrow \quad 11Q = 100 \quad \Longrightarrow \quad Q^* = \frac{100}{11},$$

and hence

$$p^* = \frac{Q^*}{2} = \frac{50}{11}.$$

Consumer surplus is the area under inverse demand above p^*:

$$CS = \int_0^{5/4} (20 - 4Q)\, dQ + \int_{5/4}^{100/11} \frac{50 - 4Q}{3}\, dQ - \frac{50}{11} \cdot \frac{100}{11}$$
$$= \frac{175}{8} + \frac{74175}{968} - \frac{5000}{121} = \frac{27675}{484}.$$

Producer surplus is the area above inverse supply and below p^*:

$$PS = \frac{1}{2} Q^* p^* = \frac{1}{2} \cdot \frac{100}{11} \cdot \frac{50}{11} = \frac{2500}{121}.$$

Thus

$$\boxed{p^* = \frac{50}{11}, \qquad Q^* = \frac{100}{11}, \qquad CS = \frac{27675}{484}, \qquad PS = \frac{2500}{121}.}$$

(c) Total-surplus comparison From Question 3.2, pre-tax total surplus is

$$TS_0 = CS_0 + PS_0 = \frac{27675}{484} + \frac{2500}{121} = \frac{3425}{44}.$$

Under the tax, we already found

$$p_b = \frac{54}{11}, \qquad Q_t = \frac{97}{11}.$$

The price received by sellers is

$$p_s = p_b - \frac{1}{2} = \frac{54}{11} - \frac{1}{2} = \frac{97}{22}.$$

Consumer surplus is

$$CS_t = \frac{26099}{484},$$

producer surplus is

$$PS_t = \frac{9409}{484},$$

and tax revenue is

$$TR = tQ_t = \frac{1}{2} \cdot \frac{97}{11} = \frac{97}{22}.$$

Therefore

$$TS_t = CS_t + PS_t + TR = \frac{26099}{484} + \frac{9409}{484} + \frac{97}{22} = \frac{1711}{22}.$$

Hence

$$DWL = TS_0 - TS_t = \frac{3425}{44} - \frac{1711}{22} = \frac{3425 - 3422}{44} = \frac{3}{44}.$$

So

$$\boxed{DWL = \frac{3}{44}.}$$

Question 4: Mala-Hotpot Excise Tax

Problem

In the mala-hotpot market, market demand is given by $D(P_b) = 1000 - 10P_b$.

The individual supply of each stall is given by $S_i(P_s) = 10P_s - 10$. Suppose there are n identical stalls in the market. As the government thinks that mala-hotpot is unhealthy, it decides to implement a per-unit excise tax of \$2.

(a) Solve for the equilibrium price and quantity in this market without any taxes (as a function of n).

(b) Solve for the equilibrium prices and quantity in this market with the per-unit tax (as a function of n).

(c) How does the incidence of tax on consumers change as the number of stalls n increases? Explain the intuition behind this.

Solution

Method 1: Aggregate supply then equate demand and supply

(a) Without tax, buyers' and sellers' prices coincide:

$$P_b = P_s = P.$$

Each stall supplies

$$q_i = 10P - 10.$$

With n identical stalls, market supply is

$$Q_S = n(10P - 10) = 10nP - 10n.$$

Market demand is

$$Q_D = 1000 - 10P.$$

Equilibrium requires

$$1000 - 10P = 10nP - 10n.$$

So

$$1000 + 10n = 10P(n + 1).$$

Hence

$$P^* = \frac{100 + n}{n + 1}.$$

Substituting into demand,

$$\begin{aligned} Q^* &= 1000 - 10\left(\frac{100+n}{n+1}\right) \\ &= \frac{1000(n+1) - 10(100+n)}{n+1} \\ &= \frac{1000n + 1000 - 1000 - 10n}{n+1} \\ &= \frac{990n}{n+1}. \end{aligned}$$

Therefore

$$\boxed{P^* = \frac{100+n}{n+1}, \qquad Q^* = \frac{990n}{n+1}.}$$

(b) Buyers' price, sellers' price, and the tax wedge With a per-unit tax of 2,

$$P_b = P_s + 2.$$

Market demand is

$$Q_D = 1000 - 10P_b,$$

and market supply is

$$Q_S = 10nP_s - 10n.$$

Equilibrium requires

$$1000 - 10(P_s + 2) = 10nP_s - 10n.$$

So

$$980 = 10(n+1)P_s - 10n,$$

hence

$$98 = (n+1)P_s - n.$$

Therefore

$$P_s = \frac{98+n}{n+1}.$$

Then

$$P_b = P_s + 2 = \frac{98+n}{n+1} + 2 = \frac{100+3n}{n+1}.$$

The equilibrium quantity is

$$Q_t = 10nP_s - 10n = 10n\left(\frac{98+n}{n+1} - 1\right) = 10n\left(\frac{97}{n+1}\right) = \frac{970n}{n+1}.$$

Hence

$$\boxed{P_s = \frac{98+n}{n+1}, \qquad P_b = \frac{100+3n}{n+1}, \qquad Q_t = \frac{970n}{n+1}.}$$

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(c) Compare the pre-tax and post-tax prices From Question 4.1,

$$P^* = \frac{100+n}{n+1}.$$

From Question 4.2,

$$P_b = \frac{100+3n}{n+1}, \qquad P_s = \frac{98+n}{n+1}.$$

Hence the amount of the tax borne by consumers is

$$P_b - P^* = \frac{100+3n}{n+1} - \frac{100+n}{n+1} = \frac{2n}{n+1}.$$

The amount borne by producers is

$$P^* - P_s = \frac{100+n}{n+1} - \frac{98+n}{n+1} = \frac{2}{n+1}.$$

These add up to the full tax:

$$\frac{2n}{n+1} + \frac{2}{n+1} = 2.$$

Therefore the consumer share of the tax is

$$\frac{P_b - P^*}{2} = \frac{n}{n+1}.$$

As n increases,

$$\frac{n}{n+1} \uparrow 1.$$

So consumers bear a larger and larger fraction of the tax, and in the limit they bear almost all of it.

Thus

Consumer incidence rises with n, and tends to the full \$2 tax as $n \to \infty$.

Method 2: Inverse supply representation

(a) Each stall's direct supply is

$$q_i = 10P_s - 10 \quad \Longrightarrow \quad P_s = 1 + \frac{q_i}{10}.$$

With n identical stalls, if total output is Q, symmetry implies each stall supplies $q_i = Q/n$. Hence the inverse market supply is

$$P = 1 + \frac{Q}{10n}.$$

The inverse demand is

$$P = 100 - \frac{Q}{10}.$$

Equating inverse demand and inverse supply,

$$100 - \frac{Q}{10} = 1 + \frac{Q}{10n}.$$

Therefore

$$99 = \frac{Q}{10} + \frac{Q}{10n} = \frac{Q(n+1)}{10n}.$$

So

$$Q^* = \frac{990n}{n+1}.$$

Then

$$P^* = 100 - \frac{Q^*}{10} = 100 - \frac{99n}{n+1} = \frac{100+n}{n+1}.$$

Thus

$$\boxed{P^* = \frac{100+n}{n+1}, \qquad Q^* = \frac{990n}{n+1}.}$$

(b) Inverse supply shifted by the tax Without tax, the inverse market supply is

$$P_s = 1 + \frac{Q}{10n}.$$

A tax of 2 raises the buyers' price needed to induce any given quantity by 2, so in buyers' price terms the taxed inverse supply is

$$P_b = 3 + \frac{Q}{10n}.$$

Inverse demand is

$$P_b = 100 - \frac{Q}{10}.$$

Equating them,

$$100 - \frac{Q}{10} = 3 + \frac{Q}{10n}.$$

Thus

$$97 = \frac{Q}{10} + \frac{Q}{10n} = \frac{Q(n+1)}{10n},$$

so

$$Q_t = \frac{970n}{n+1}.$$

Then

$$P_b = 100 - \frac{Q_t}{10} = 100 - \frac{97n}{n+1} = \frac{100+3n}{n+1},$$

and

$$P_s = P_b - 2 = \frac{98+n}{n+1}.$$

Therefore

$$\boxed{P_s = \frac{98+n}{n+1}, \qquad P_b = \frac{100+3n}{n+1}, \qquad Q_t = \frac{970n}{n+1}.}$$

(c) Elasticity intuition via market supply The aggregate supply curve is

$$Q_S = 10nP_s - 10n.$$

In inverse form,

$$P_s = 1 + \frac{Q}{10n}.$$

As n increases, the coefficient $1/(10n)$ becomes smaller, so the inverse supply curve becomes flatter. Equivalently, aggregate supply becomes more elastic.

When supply is very elastic, firms are unwilling to absorb much of the tax through a lower net-of-tax price. Instead, most of the tax is passed on to consumers in the form of a higher buyers' price. That is exactly what the formula

$$P_b - P^* = \frac{2n}{n+1}$$

shows.

Hence

$$\boxed{\text{More stalls } \Rightarrow \text{ more elastic market supply } \Rightarrow \text{ higher consumer tax incidence.}}$$

Sample Question 1: Good X with Heterogeneous Consumers and Firms

Problem

In the market for good X, which has a price of p, there are 160 consumers with an identical income of \$100. There are two types of consumers, Type 1 and Type 2, each with different preferences.
Type 1s have preferences

$$U_1(x) = 16x - 4x^2 + m,$$

where x is the quantity of good x purchased and m is their leftover cash holdings. Type 2s have preferences

$$U_2(x) = 8x - 8x^2 + m.$$

There are 80 consumers of Type 1 and 80 consumers of Type 2.

(a) Derive the individual demand curves for Type 1 and Type 2 consumers. **(6 marks)**

(b) Derive the aggregate demand curve for good X. **(5 marks)**

In the market for good X, there are also a total of n identical firms who are profit maximisers. Each of these firms has a cost function of$3x^2$.

(c) Maximise the profits of each firm ($px - 3x^2$) to derive the individual supply curve. Subsequently derive the aggregate supply curve for good X. **(5 marks)**

(d) How does the market's elasticity of supply change with the number of firms in the market? Explain. **(4 marks)**

Suppose that $n = 72$, i.e. there are 72 firms in the market.

(e) Draw out the aggregate demand and supply curves in the market, calculating and indicating the exact market equilibrium on your figure. **(7 marks)**

(f) What is the amount of consumer and producer surplus in the market? **(5 marks)**

Suppose that the government now imposes a per unit tax of \$6 on producers.

(g) Illustrate in a diagram, as accurately as possible, the effect of the tax, indicating i) the deadweight welfare loss, and ii) the incidence of the tax burden on consumers and producers. **(8 marks)**

Solution

Method 1: Direct optimisation, aggregation, and equilibrium solving

(a) For Type 1, since leftover cash is

$$m = 100 - px,$$

utility becomes

$$U_1(x) = 16x - 4x^2 + 100 - px.$$

Differentiate:

$$\frac{dU_1}{dx} = 16 - 8x - p.$$

Setting the first-order condition equal to zero,

$$16 - 8x - p = 0 \quad \Longrightarrow \quad x = \frac{16 - p}{8} = 2 - \frac{p}{8}.$$

Because

$$\frac{d^2U_1}{dx^2} = -8 < 0,$$

this is the maximiser whenever it is nonnegative. Hence

$$x_1(p) = \begin{cases} 2 - \dfrac{p}{8}, & 0 \le p \le 16, \\ 0, & p > 16. \end{cases}$$

For Type 2,

$$U_2(x) = 8x - 8x^2 + 100 - px.$$

Differentiate:

$$\frac{dU_2}{dx} = 8 - 16x - p.$$

Setting this equal to zero,

$$8 - 16x - p = 0 \quad \Longrightarrow \quad x = \frac{8 - p}{16} = \frac{1}{2} - \frac{p}{16}.$$

Since

$$\frac{d^2U_2}{dx^2} = -16 < 0,$$

this is the maximiser whenever nonnegative. Therefore

$$x_2(p) = \begin{cases} \dfrac{1}{2} - \dfrac{p}{16}, & 0 \le p \le 8, \\ 0, & p > 8. \end{cases}$$

Thus

$$\boxed{x_1(p) = \begin{cases} 2 - \dfrac{p}{8}, & 0 \le p \le 16, \\ 0, & p > 16, \end{cases} \qquad x_2(p) = \begin{cases} \dfrac{1}{2} - \dfrac{p}{16}, & 0 \le p \le 8, \\ 0, & p > 8. \end{cases}}$$

(b) There are 80 consumers of each type, so aggregate demand is

$$Q_D(p) = 80x_1(p) + 80x_2(p).$$

For $0 \leq p \leq 8$, both types buy:

$$\begin{aligned} Q_D(p) &= 80\left(2 - \frac{p}{8}\right) + 80\left(\frac{1}{2} - \frac{p}{16}\right) \\ &= 160 - 10p + 40 - 5p \\ &= 200 - 15p. \end{aligned}$$

For $8 < p \leq 16$, only Type 1 buys:

$$Q_D(p) = 80\left(2 - \frac{p}{8}\right) = 160 - 10p.$$

For $p > 16$, nobody buys:

$$Q_D(p) = 0.$$

Hence

$$\boxed{Q_D(p) = \begin{cases} 200 - 15p, & 0 \leq p \leq 8, \\ 160 - 10p, & 8 < p \leq 16, \\ 0, & p > 16. \end{cases}}$$

(c) Each firm solves

$$\max_{x \geq 0} \ \pi(x) = px - 3x^2.$$

Differentiate:

$$\frac{d\pi}{dx} = p - 6x.$$

Setting this equal to zero,

$$p - 6x = 0 \quad \Longrightarrow \quad x = \frac{p}{6}.$$

Since

$$\frac{d^2\pi}{dx^2} = -6 < 0,$$

this is the profit-maximising quantity. Therefore the individual supply curve is

$$\boxed{x_s(p) = \frac{p}{6}.}$$

With n identical firms, aggregate supply is

$$Q_S(p) = n\frac{p}{6}.$$

So

$$\boxed{Q_S(p) = \frac{n}{6}p.}$$

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(d) Since

$$Q_S(p) = \frac{n}{6}p,$$

we have

$$\frac{dQ_S}{dp} = \frac{n}{6}.$$

The price elasticity of supply is

$$\varepsilon_S = \frac{dQ_S}{dp} \cdot \frac{p}{Q_S(p)} = \frac{n}{6} \cdot \frac{p}{(n/6)p} = 1.$$

Hence the market's elasticity of supply does not change with the number of firms:

$$\boxed{\varepsilon_S = 1 \text{ for all } p > 0.}$$

The number of firms changes the scale of market supply, but not its elasticity.

(e) Set $n = 72$. Then market supply is

$$Q_S(p) = \frac{72}{6}p = 12p.$$

Since the equilibrium price will turn out to be below 8, the relevant demand branch is

$$Q_D(p) = 200 - 15p.$$

Equating demand and supply,

$$200 - 15p = 12p.$$

Hence

$$27p = 200 \quad \Longrightarrow \quad p^* = \frac{200}{27}.$$

Therefore

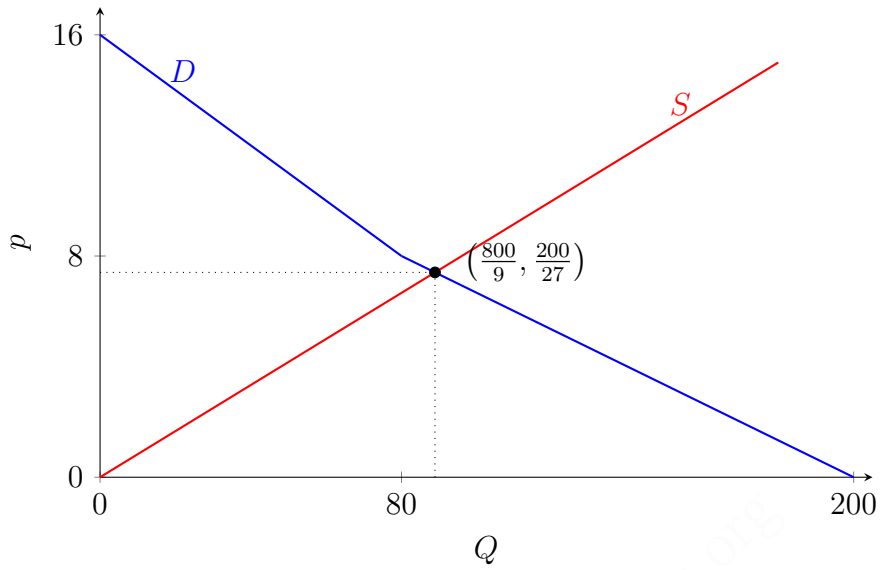
$$Q^* = 12p^* = \frac{800}{9}.$$

Thus

$$\boxed{p^* = \frac{200}{27}, \qquad Q^* = \frac{800}{9}.}$$

A suitable graph is:

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(f) At the equilibrium price $p^* = \frac{200}{27}$, Type 1 buys

$$x_1^* = 2 - \frac{1}{8} \cdot \frac{200}{27} = 2 - \frac{25}{27} = \frac{29}{27},$$

while Type 2 buys

$$x_2^* = \frac{1}{2} - \frac{1}{16} \cdot \frac{200}{27} = \frac{1}{2} - \frac{25}{54} = \frac{1}{27}.$$

Type 1 consumer surplus per consumer is

$$CS_1^{\text{ind}} = \frac{1}{2}\left(16 - \frac{200}{27}\right)\frac{29}{27} = \frac{3364}{729}.$$

Across 80 Type 1 consumers:

$$CS_1 = 80 \cdot \frac{3364}{729} = \frac{269120}{729}.$$

Type 2 consumer surplus per consumer is

$$CS_2^{\text{ind}} = \frac{1}{2}\left(8 - \frac{200}{27}\right)\frac{1}{27} = \frac{8}{729}.$$

Across 80 Type 2 consumers:

$$CS_2 = 80 \cdot \frac{8}{729} = \frac{640}{729}.$$

Therefore

$$CS = CS_1 + CS_2 = \frac{269760}{729} = \frac{89920}{243}.$$

So

$$\boxed{CS = \frac{89920}{243}.}$$

Producer surplus is

$$PS = \frac{1}{2}p^*Q^* = \frac{1}{2} \cdot \frac{200}{27} \cdot \frac{800}{9} = \frac{80000}{243}.$$

Hence

$$\boxed{PS = \frac{80000}{243}.}$$

(g) A per-unit tax of 6 on producers implies

$$p_b = p_s + 6.$$

Since aggregate supply before tax is

$$Q_S = 12p_s,$$

aggregate supply in terms of the buyers' price is

$$Q_S^t(p_b) = 12(p_b - 6) = 12p_b - 72.$$

After tax the buyers' price exceeds 8, so only Type 1 remains active. Hence the relevant demand branch is

$$Q_D(p_b) = 160 - 10p_b.$$

Equating demand and taxed supply,

$$160 - 10p_b = 12p_b - 72.$$

Therefore

$$232 = 22p_b \quad \Longrightarrow \quad p_b = \frac{116}{11}.$$

The taxed quantity is

$$Q_t = 160 - 10p_b = \frac{600}{11},$$

and the sellers' net price is

$$p_s = p_b - 6 = \frac{50}{11}.$$

Thus

$$\boxed{p_b = \frac{116}{11}, \qquad p_s = \frac{50}{11}, \qquad Q_t = \frac{600}{11}.}$$

The consumer burden is

$$p_b - p^* = \frac{116}{11} - \frac{200}{27} = \frac{932}{297},$$

while the producer burden is

$$p^* - p_s = \frac{200}{27} - \frac{50}{11} = \frac{850}{297}.$$

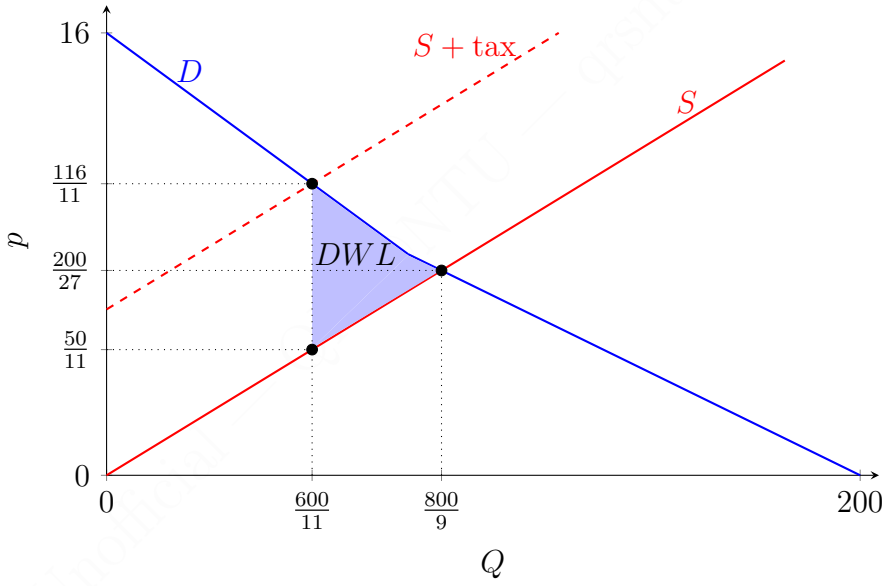
The deadweight loss is

$$DWL = \frac{1}{2} \cdot 6 \left(\frac{800}{9} - \frac{600}{11} \right) = \frac{3400}{33}.$$

Hence

$$\boxed{\text{consumer burden} = \frac{932}{297}, \quad \text{producer burden} = \frac{850}{297}, \quad DWL = \frac{3400}{33}.}$$

A suitable tax diagram is:



Method 2: Inverse-demand / inverse-supply and welfare geometry

(a) The individual demand curves can be written compactly as

$$x_1(p) = \left(2 - \frac{p}{8}\right)_+, \qquad x_2(p) = \left(\frac{1}{2} - \frac{p}{16}\right)_+,$$

where $(z)_+ = \max\{z, 0\}$. Equivalently,

$$\boxed{x_1(p) = \begin{cases} 2 - \frac{p}{8}, & 0 \le p \le 16, \\ 0, & p > 16, \end{cases} \qquad x_2(p) = \begin{cases} \frac{1}{2} - \frac{p}{16}, & 0 \le p \le 8, \\ 0, & p > 8. \end{cases}}$$

(b) Aggregate demand is

$$Q_D(p) = 80\left(2 - \frac{p}{8}\right)_+ + 80\left(\frac{1}{2} - \frac{p}{16}\right)_+.$$

Inverse demand is therefore piecewise:

$$p_D(Q) = \begin{cases} 16 - \frac{Q}{10}, & 0 \le Q \le 80, \\ \frac{200 - Q}{15}, & 80 \le Q \le 200. \end{cases}$$

Hence

$$\boxed{p_D(Q) = \begin{cases} 16 - \frac{Q}{10}, & 0 \le Q \le 80, \\ \frac{200 - Q}{15}, & 80 \le Q \le 200. \end{cases}}$$

(c) Each firm has cost $3x^2$, so marginal cost is

$$MC(x) = 6x.$$

Competitive supply satisfies $p = MC$, so

$$x_s(p) = \frac{p}{6}.$$

Aggregating across n firms gives

$$Q_S(p) = \frac{n}{6}p, \qquad p_S(Q) = \frac{6Q}{n}.$$

Therefore

$$\boxed{Q_S(p) = \frac{n}{6}p, \qquad p_S(Q) = \frac{6Q}{n}.}$$

(d) Because $Q_S(p)$ is proportional to p, its elasticity is identically 1:

$$\varepsilon_S = \frac{dQ_S}{dp} \cdot \frac{p}{Q_S} = 1.$$

Hence

$$\boxed{\varepsilon_S = 1.}$$

(e) For $n = 72$,

$$p_S(Q) = \frac{Q}{12}.$$

Solve using the lower branch of inverse demand:

$$\frac{200 - Q}{15} = \frac{Q}{12}.$$

Therefore

$$12(200 - Q) = 15Q \quad \Longrightarrow \quad 2400 = 27Q \quad \Longrightarrow \quad Q^* = \frac{800}{9}.$$

Then

$$p^* = \frac{Q^*}{12} = \frac{200}{27}.$$

Thus

$$\boxed{Q^* = \frac{800}{9}, \qquad p^* = \frac{200}{27}.}$$

(f) Consumer surplus is the area under inverse demand above price:

$$\begin{aligned} CS &= \int_0^{80} \left(16 - \frac{Q}{10}\right) dQ + \int_{80}^{800/9} \left(\frac{200 - Q}{15}\right) dQ - \frac{200}{27} \cdot \frac{800}{9} \\ &= \frac{89920}{243}. \end{aligned}$$

Producer surplus is

$$PS = \frac{1}{2} Q^* p^* = \frac{1}{2} \cdot \frac{800}{9} \cdot \frac{200}{27} = \frac{80000}{243}.$$

Hence

$$\boxed{CS = \frac{89920}{243}, \qquad PS = \frac{80000}{243}.}$$

(g) A tax of 6 on producers shifts inverse supply vertically upward by 6:

$$p_{S,t}(Q) = \frac{Q}{12} + 6.$$

After tax, the relevant demand branch is the upper one:

$$16 - \frac{Q}{10} = \frac{Q}{12} + 6.$$

Solving,

$$10 = \frac{11Q}{60} \quad \Longrightarrow \quad Q_t = \frac{600}{11}.$$

Therefore

$$p_b = 16 - \frac{Q_t}{10} = \frac{116}{11}, \qquad p_s = p_b - 6 = \frac{50}{11}.$$

The deadweight loss is

$$DWL = \frac{1}{2} \cdot 6 \left(\frac{800}{9} - \frac{600}{11} \right) = \frac{3400}{33}.$$

Thus

$$\boxed{Q_t = \frac{600}{11}, \qquad p_b = \frac{116}{11}, \qquad p_s = \frac{50}{11}, \qquad DWL = \frac{3400}{33}.}$$

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Method 3: Comparative-statics / elasticity and regime-switch method

(a) Write individual demands as

$$x_1(p) = \left(2 - \frac{p}{8}\right)_+, \qquad x_2(p) = \left(\frac{1}{2} - \frac{p}{16}\right)_+.$$

The cutoffs are therefore

$$p = 16 \quad \text{for Type 1}, \qquad p = 8 \quad \text{for Type 2}.$$

So Type 2 exits first as price rises.

(b) Aggregate demand is

$$Q_D(p) = 80\left(2 - \frac{p}{8}\right)_+ + 80\left(\frac{1}{2} - \frac{p}{16}\right)_+,$$

which gives the two active regimes

$$Q_D(p) = 200 - 15p \quad (0 \le p \le 8), \qquad Q_D(p) = 160 - 10p \quad (8 < p \le 16).$$

The structural point is that the demand slope changes when Type 2 exits.

(c) Supply is

$$Q_S(p) = \frac{n}{6}p,$$

so changing n rotates market supply outward through the origin.

(d) Even though changing n changes the level of market supply, the elasticity remains

$$\varepsilon_S = 1.$$

Thus

$$\boxed{\text{more firms } \Rightarrow \text{ more supply at each price, but the same elasticity.}}$$

(e) To determine which demand branch is relevant, compare the supply quantity at $p = 8$:

$$Q_S(8) = \frac{n}{6} \cdot 8 = \frac{4n}{3}.$$

If both types are active, then the branch switch occurs at the market kink $(Q, p) = (80, 8)$. Hence:

$$\frac{4n}{3} \gtrless 80 \quad \Longleftrightarrow \quad n \gtrless 60.$$

So:

- if $n > 60$, equilibrium lies on the lower branch $Q_D = 200 - 15p$;
- if $n < 60$, equilibrium lies on the upper branch $Q_D = 160 - 10p$;

- if $n = 60$, equilibrium is exactly at the kink.

Since $n = 72 > 60$, we use

$$200 - 15p = 12p,$$

which yields

$$\boxed{p^* = \frac{200}{27}, \qquad Q^* = \frac{800}{9}.}$$

(f) Since $Q^* = \frac{800}{9} > 80$, both consumer types remain active in the no-tax equilibrium, but Type 2 buys only a very small amount:

$$x_2^* = \frac{1}{27}.$$

This already suggests that a sufficiently strong tax can push Type 2 completely out of the market.

Using the area formulas on the aggregate curves,

$$\boxed{CS = \frac{89920}{243}, \qquad PS = \frac{80000}{243}.}$$

(g) After a producer tax $t = 6$, buyer-price supply becomes

$$Q_S^t(p_b) = \frac{n}{6}(p_b - 6).$$

For $n = 72$,

$$Q_S^t(p_b) = 12p_b - 72.$$

Check what happens at the demand cutoff $p_b = 8$:

$$Q_S^t(8) = 24, \qquad Q_D(8) = 80.$$

Since taxed supply at $p_b = 8$ is far below demand at the kink, the post-tax equilibrium must occur on the upper branch where only Type 1 remains active. Hence solve

$$160 - 10p_b = 12p_b - 72.$$

Therefore

$$\boxed{p_b = \frac{116}{11}, \qquad p_s = \frac{50}{11}, \qquad Q_t = \frac{600}{11}.}$$

The key comparative-statics insight is that the tax does more than just reduce quantity: it changes the active set of consumers. Type 2 exits completely after tax. The resulting deadweight loss is

$$\boxed{DWL = \frac{3400}{33}.}$$

Marking scheme (40 marks)

(a) **(6 marks)** Individual demand curves

- 2 marks: Correct setup of the Type 1 utility-maximisation problem, including substitution of leftover cash $m = 100 - px$ or an equivalent formulation.
- 2 marks: Correct derivation of the Type 1 demand curve, including the non-negativity condition.
- 2 marks: Correct derivation of the Type 2 demand curve, including the non-negativity condition.

(b) **(5 marks)** Aggregate demand curve

- 2 marks: Correct identification of the relevant price regions and which consumer types are active in each region.
- 2 marks: Correct piecewise aggregation of the two individual demand curves.
- 1 mark: Correct final aggregate demand curve, including the kink / cutoff structure.

(c) **(5 marks)** Individual and aggregate supply

- 2 marks: Correct setup of the firm's profit-maximisation problem.
- 1 mark: Correct individual supply curve.
- 2 marks: Correct aggregate supply curve for n identical firms.

(d) **(4 marks)** Market elasticity of supply

- 2 marks: Correct elasticity result, including whether it changes with the number of firms.
- 2 marks: Clear economic explanation of why the elasticity does or does not change as n varies.

(e) **(7 marks)** Equilibrium for $n = 72$

- 2 marks: Correct substitution of $n = 72$ into the aggregate supply curve.
- 2 marks: Correct identification of the relevant branch of aggregate demand.
- 2 marks: Correct equilibrium price and quantity.
- 1 mark: Diagram with demand, supply, and the exact equilibrium correctly indicated.

(f) **(5 marks)** Consumer and producer surplus

- 2 marks: Correct computation of consumer surplus.
- 2 marks: Correct computation of producer surplus.
- 1 mark: Correct final expressions / values, consistent with the equilibrium from part (e).

(g) **(8 marks)** Per-unit tax on producers

- 2 marks: Correct setup of the post-tax equilibrium, including the tax wedge between buyers' and sellers' prices.
- 2 marks: Correct taxed prices and quantity, or a fully correct equivalent derivation.
- 2 marks: Correct deadweight welfare loss, or a correct DWL triangle with correct dimensions.
- 2 marks: Diagram and explanation correctly showing the incidence of the tax burden on consumers and producers.

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Sample Question 2: Teddy Bears

Problem

Consider the market for teddy bears. [1] There are n firms with *individual* supply curves given by

$$S(p) = 4p - 10.$$

The *market* demand curve is given by

$$D(p) = 1000 - 80p.$$

(a) Derive the market supply curve in terms of n. **(5 marks)**

(b) Suppose there is a per-unit tax of \$5 on the *consumers*. For $n = 10$, draw out the market supply and demand curves *with* and *without* tax, indicating and explaining the deadweight welfare loss from the tax. **(9 marks)**

(c) For the same per-unit tax on consumers, calculate the deadweight welfare loss as a function of the number of producers in the market n. **(10 marks)**

(d) How does the deadweight welfare loss change with the number of producers? Explain the intuition behind this. **(6 marks)**

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[1] There appears to be a likely typo in the handout. If the individual supply curve is written as $S(p) = 4p - 100$, then positive supply requires $p \geq 25$, while market demand $D(p) = 1000 - 80p$ falls to zero at $p = 12.5$. Hence there is no interior trade equilibrium, so the later tax and deadweight-loss parts become degenerate. The intended version is very likely $S(p) = 4p - 10$, which yields a standard non-degenerate competitive-equilibrium and welfare-analysis problem.

Solution

Method 1: Direct aggregation and tax-wedge algebra

(a) Each firm supplies

$$q_i(p) = 4p - 10.$$

With n identical firms, market supply is

$$Q_S(p) = n(4p - 10) = 4np - 10n.$$

Therefore

$$\boxed{Q_S(p) = 4np - 10n.}$$

(b) For $n = 10$, the market supply curve is

$$Q_S(p) = 40p - 100.$$

Demand is

$$Q_D(p) = 1000 - 80p.$$

Before tax, equilibrium satisfies

$$1000 - 80p = 40p - 100.$$

Hence

$$1100 = 120p \quad \Longrightarrow \quad p^* = \frac{55}{6}.$$

Then

$$Q^* = 1000 - 80p^* = 1000 - 80 \cdot \frac{55}{6} = \frac{800}{3}.$$

Thus

$$\boxed{p^* = \frac{55}{6}, \qquad Q^* = \frac{800}{3}.}$$

Now impose a per-unit tax of 5 on consumers. Let p_b be the buyers' price and p_s the sellers' price. Then

$$p_b = p_s + 5.$$

Demand is

$$Q_D = 1000 - 80p_b,$$

while supply depends on the sellers' price:

$$Q_S = 40p_s - 100.$$

Substitute $p_s = p_b - 5$:

$$Q_S = 40(p_b - 5) - 100 = 40p_b - 300.$$

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Equate demand and supply:

$$1000 - 80p_b = 40p_b - 300.$$

Hence

$$1300 = 120p_b \quad \Longrightarrow \quad p_b = \frac{65}{6}.$$

Therefore

$$p_s = p_b - 5 = \frac{35}{6}.$$

The taxed quantity is

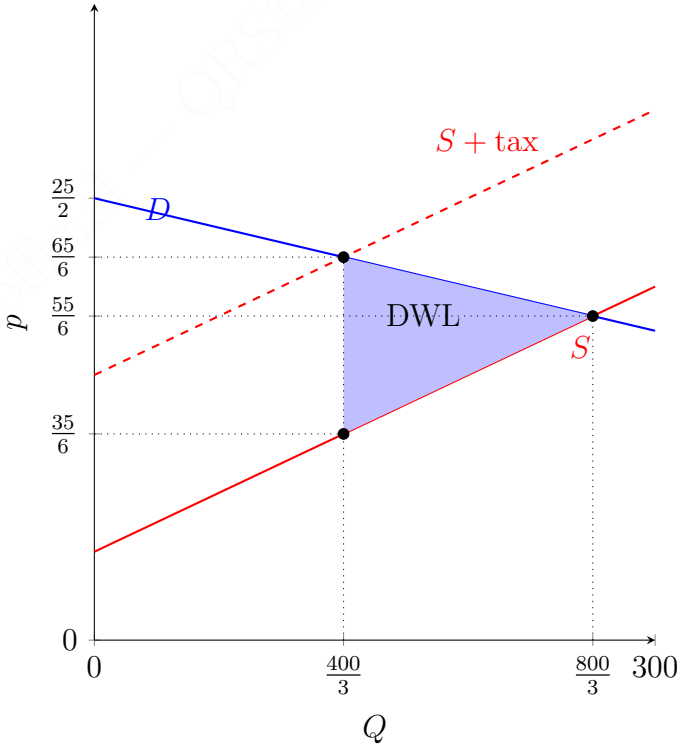
$$Q_t = 1000 - 80p_b = 1000 - 80 \cdot \frac{65}{6} = \frac{400}{3}.$$

Thus

$$\boxed{p_b = \frac{65}{6}, \qquad p_s = \frac{35}{6}, \qquad Q_t = \frac{400}{3}.}$$

The deadweight loss is $DWL = \frac{1}{2} \cdot 5 \left(\frac{800}{3} - \frac{400}{3} \right) = \frac{1000}{3}$. Hence $\boxed{DWL = \frac{1000}{3}.}$

A suitable graph[2] is:



[2]Although the solution represents the consumer tax as an upward shift of supply, an equivalent and often clearer representation is a downward shift of demand, which more directly reflects the tax being levied on consumers. Both approaches impose the same wedge $p_b - p_s = t$ and yield identical equilibrium outcomes.

(c) For general n, before tax:

$$1000 - 80p = 4np - 10n.$$

So

$$1000 + 10n = (80 + 4n)p \quad \Longrightarrow \quad p^*(n) = \frac{1000 + 10n}{80 + 4n}.$$

The corresponding quantity is

$$Q_0(n) = 1000 - 80p^*(n) = \frac{800n}{n + 20}.$$

After a consumer tax of 5:

$$p_b = p_s + 5.$$

Since

$$Q_S = 4np_s - 10n,$$

in terms of the buyers' price we have

$$Q_S = 4n(p_b - 5) - 10n = 4np_b - 30n.$$

Equating with demand:

$$1000 - 80p_b = 4np_b - 30n.$$

Therefore

$$1000 + 30n = (80 + 4n)p_b,$$

and the taxed quantity is

$$Q_t(n) = 1000 - 80p_b = \frac{400n}{n + 20}.$$

Thus

$$DWL(n) = \frac{1}{2} \cdot 5\big(Q_0(n) - Q_t(n)\big) = \frac{1}{2} \cdot 5 \left(\frac{800n}{n + 20} - \frac{400n}{n + 20} \right).$$

Hence

$$\boxed{DWL(n) = \frac{1000n}{n + 20}.}$$

(d) Since

$$DWL(n) = \frac{1000n}{n + 20},$$

we differentiate:

$$\frac{d}{dn} DWL(n) = \frac{20000}{(n + 20)^2} > 0.$$

Therefore deadweight loss increases with the number of producers. Moreover,

$$\lim_{n \to \infty} DWL(n) = 1000.$$

So

$$\boxed{\text{DWL increases with } n \text{ and tends to 1000 as } n \to \infty.}$$

Intuitively, more firms make aggregate supply more elastic, so a given tax causes a larger quantity distortion and therefore a larger deadweight loss.

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Method 2: Inverse-demand / inverse-supply and welfare geometry

(a) Since

$$Q_S = 4np - 10n,$$

the inverse supply curve is

$$p_S(Q) = \frac{Q + 10n}{4n} = \frac{Q}{4n} + \frac{5}{2}.$$

Thus

$$\boxed{Q_S(p) = 4np - 10n, \qquad p_S(Q) = \frac{Q}{4n} + \frac{5}{2}.}$$

(b) Inverse demand is

$$p_D(Q) = \frac{1000 - Q}{80} = \frac{25}{2} - \frac{Q}{80}.$$

For $n = 10$, inverse supply is

$$p_S(Q) = \frac{Q}{40} + \frac{5}{2}.$$

Before tax:

$$\frac{25}{2} - \frac{Q}{80} = \frac{Q}{40} + \frac{5}{2}.$$

Thus

$$10 = \frac{3Q}{80} \quad \Longrightarrow \quad Q^* = \frac{800}{3}.$$

Then

$$p^* = \frac{25}{2} - \frac{1}{80} \cdot \frac{800}{3} = \frac{55}{6}.$$

With a consumer tax of 5, the buyers' supply curve shifts up by 5:

$$p_{S,t}(Q) = \frac{Q}{40} + \frac{15}{2}.$$

Equating with inverse demand:

$$\frac{25}{2} - \frac{Q}{80} = \frac{Q}{40} + \frac{15}{2}.$$

Therefore

$$5 = \frac{3Q}{80} \quad \Longrightarrow \quad Q_t = \frac{400}{3}.$$

The buyers' price is

$$p_b = \frac{25}{2} - \frac{1}{80} \cdot \frac{400}{3} = \frac{65}{6},$$

and the sellers' price is

$$p_s = p_b - 5 = \frac{35}{6}.$$

Hence

$$\boxed{Q^* = \frac{800}{3}, \quad p^* = \frac{55}{6}, \quad Q_t = \frac{400}{3}, \quad p_b = \frac{65}{6}, \quad p_s = \frac{35}{6}.}$$

The deadweight loss is

$$DWL = \frac{1}{2} \cdot 5 \left(\frac{800}{3} - \frac{400}{3} \right) = \frac{1000}{3}.$$

Therefore

$$\boxed{DWL = \frac{1000}{3}.}$$

(c) For general n, inverse supply is

$$p_S(Q) = \frac{Q}{4n} + \frac{5}{2},$$

while inverse demand is

$$p_D(Q) = \frac{25}{2} - \frac{Q}{80}.$$

Before tax:

$$\frac{25}{2} - \frac{Q}{80} = \frac{Q}{4n} + \frac{5}{2}.$$

Hence

$$10 = Q \left(\frac{1}{80} + \frac{1}{4n} \right) = Q \frac{n+20}{80n},$$

so

$$Q_0(n) = \frac{800n}{n+20}.$$

With tax $t = 5$, buyer-side inverse supply is

$$p_{S,t}(Q) = \frac{Q}{4n} + \frac{15}{2}.$$

Therefore

$$\frac{25}{2} - \frac{Q}{80} = \frac{Q}{4n} + \frac{15}{2} \quad \Longrightarrow \quad 5 = Q \frac{n+20}{80n},$$

so

$$Q_t(n) = \frac{400n}{n+20}.$$

Hence

$$DWL(n) = \frac{1}{2} \cdot 5 \left(Q_0(n) - Q_t(n) \right) = \frac{1}{2} \cdot 5 \cdot \frac{400n}{n+20} = \frac{1000n}{n+20}.$$

Therefore

$$\boxed{DWL(n) = \frac{1000n}{n+20}.}$$

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(d) Since the inverse supply slope is

$$\frac{dp_S}{dQ} = \frac{1}{4n},$$

it becomes flatter as n rises. Thus supply becomes more elastic, the tax causes a larger reduction in quantity, and deadweight loss rises. Therefore

More producers $\Rightarrow$ more elastic supply $\Rightarrow$ larger DWL.

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Method 3: Concise olympiad-style tax-parameter method

(a) Market supply is

$$\boxed{Q_S(p) = 4np - 10n.}$$

Equivalently,

$$\boxed{p_S(Q) = \frac{Q}{4n} + \frac{5}{2}.}$$

(b) For $n = 10$, inverse demand and supply are

$$p_D(Q) = \frac{25}{2} - \frac{Q}{80}, \qquad p_S(Q) = \frac{Q}{40} + \frac{5}{2}.$$

Introduce a general consumer tax t. Then buyer-side inverse supply becomes

$$p_{S,t}(Q) = \frac{Q}{40} + \frac{5}{2} + t.$$

Equilibrium is defined by

$$\frac{25}{2} - \frac{Q}{80} = \frac{Q}{40} + \frac{5}{2} + t.$$

Rearranging,

$$10 - t = \frac{3Q}{80}.$$

So

$$Q_t = \frac{80}{3}(10 - t).$$

Therefore:

$$Q_0 = \frac{80}{3}(10) = \frac{800}{3}, \qquad Q_5 = \frac{80}{3}(5) = \frac{400}{3}.$$

Then

$$p^* = \frac{25}{2} - \frac{1}{80} \cdot \frac{800}{3} = \frac{55}{6},$$

$$p_b = \frac{25}{2} - \frac{1}{80} \cdot \frac{400}{3} = \frac{65}{6}, \qquad p_s = p_b - 5 = \frac{35}{6}.$$

Therefore

$$\boxed{p^* = \frac{55}{6}, \quad Q^* = \frac{800}{3}, \quad p_b = \frac{65}{6}, \quad p_s = \frac{35}{6}, \quad Q_t = \frac{400}{3}.}$$

The deadweight loss follows immediately:

$$DWL = \frac{1}{2} \cdot 5 \left(\frac{800}{3} - \frac{400}{3} \right) = \frac{1000}{3}.$$

Hence

$$\boxed{DWL = \frac{1000}{3}.}$$

(c) For general n, buyer-side inverse supply with tax t is

$$p_{S,t}(Q) = \frac{Q}{4n} + \frac{5}{2} + t.$$

Equilibrium solves

$$\frac{25}{2} - \frac{Q}{80} = \frac{Q}{4n} + \frac{5}{2} + t.$$

Hence

$$10 - t = Q\left(\frac{1}{80} + \frac{1}{4n}\right) = Q\frac{n+20}{80n}.$$

So the equilibrium quantity under tax t is

$$\boxed{Q_t(n) = \frac{80n(10-t)}{n+20}.}$$

In particular,

$$Q_0(n) = \frac{800n}{n+20}, \qquad Q_5(n) = \frac{400n}{n+20}.$$

Therefore

$$DWL(n) = \frac{1}{2} \cdot 5\left(Q_0(n) - Q_5(n)\right) = \frac{1}{2} \cdot 5 \cdot \frac{400n}{n+20}.$$

Thus

$$\boxed{DWL(n) = \frac{1000n}{n+20}.}$$

(d) The formula

$$DWL(n) = \frac{1000n}{n+20}$$

is increasing in n, because

$$\frac{d}{dn}DWL(n) = \frac{20000}{(n+20)^2} > 0.$$

So

$$\boxed{\text{DWL increases with } n.}$$

This method is structurally concise because one solves the whole tax family at once via the parameter t, then specialises to $t = 0$ and $t = 5$.

Marking scheme (30 marks)

(a) **(5 marks)** Market supply curve

- 2 marks: Correct aggregation of the individual supply curves across n firms.
- 2 marks: Correct simplified market supply curve in terms of n.
- 1 mark: Correct treatment of the economically relevant domain / interpretation of the supply curve.

(b) **(9 marks)** Tax analysis for $n = 10$

- 2 marks: Correct market supply curve for $n = 10$.
- 2 marks: Correct no-tax equilibrium analysis and/or correct identification of the no-tax market outcome.
- 2 marks: Correct treatment of the consumer tax wedge and post-tax equilibrium analysis.
- 2 marks: Correct deadweight welfare loss conclusion, including a correct explanation.
- 1 mark: Diagram clearly showing the relevant demand and supply curves, with and without tax, together with the relevant equilibrium information.

(c) **(10 marks)** Deadweight welfare loss as a function of n

- 3 marks: Correct no-tax equilibrium analysis in terms of n, or a correct general argument about the pre-tax market outcome.
- 3 marks: Correct post-tax equilibrium analysis in terms of n, or a correct general argument about the post-tax market outcome.
- 3 marks: Correct derivation of deadweight welfare loss as a function of n.
- 1 mark: Correct final simplified expression.

(d) **(6 marks)** Comparative statics of deadweight welfare loss

- 2 marks: Correct statement of how deadweight welfare loss changes with the number of producers.
- 2 marks: Correct supporting reasoning using the shape / elasticity / position of market supply.
- 2 marks: Clear economic intuition connecting the number of producers to the quantity distortion and welfare loss.