Sem 2 AY2122 MH1101 Calculus II Test (1 hour)

Name: ..... **Matriculation Number:** .....

Tutorial Group: .....

Question 1 [16 marks]

(a) Calculate the derivative $\dfrac{d}{dx}\displaystyle\int_x^{x^3+\pi x} e^{\sin t}\,dt$. Express your final answer in terms of x.

(b) Write the following limit as a definite integral (do not evaluate it).

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{4i+n}{2i^2+n^2}$$

Answer.

More space for answers.

Question 2 [14 marks]

(a) Let R be the region bounded by the curves $y^2 = 9x$, $x + y = 10$ and the y-axis. Compute the volume of the solid formed by revolving R about the line $y = 10$.

(b) Describe a solid of revolution (by identifying the region on the xy-plane to be rotated, and the axis of rotation) whose volume is given by the definite integral

$$\int_0^5 2\pi(5-y)^2\,dy.$$

Answer.

More space for answers.

More space for answers.

Question 3 [20 marks]

(a) Evaluate the following integrals:

(i) $\displaystyle\int \tan^2(3x)\sec^6(3x)\,dx$

(ii) $\displaystyle\int e^{2x}\sqrt{5+2e^x}\,dx$

(iii) $\displaystyle\int \sec^3 x\,dx$

(b) Assume that the following integrals exist. Determine whether the statement/equation below is (always) true or false. Justify your answer. Hint: Use substitution.

$$\int_0^\infty \frac{x^2}{x^4+1}\,dx = \frac{1}{2}\int_0^\infty \frac{x^2+1}{x^4+1}\,dx.$$

Answer.

More space for answers.

More space for answers.

More space for answers.

Formulae Table

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C$

$\int k\,dx = kx + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \csc x \cot x\,dx = -\csc x + C$

$\int \tan x\,dx = \ln|\sec x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin 2x = 2\sin x \cos x$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$\sin A \cos B = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right)$$

$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

$$\cos A \cos B = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

– End of Test –