Nanyang Technological University
SPMS/Division of Mathematical Sciences

2022/2023 Semester 2 **MH1201** **Mid-term**
24 Feb 2023 TIME ALLOWED: 60 MINUTES

Name:	**Matriculation Number**:

Question	1	2	3	Total
Marks				

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **THREE (3)** questions and comprises **SEVEN (7)** printed pages, including the cover page.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. You may write on the opposite page or any page if there is not enough space.
4. This is a **CLOSED BOOK** exam.
5. You may use calculators.

QUESTION 1. **(15 Marks)**

Let $\boldsymbol{T} : \mathcal{M}_{2\times 2}(\mathbb{R}) \to \mathbb{R}^4$ be the map defined by

$$\boldsymbol{T}\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) = \left(\, a_{11} + a_{12}, \; a_{21} + a_{22}, \; a_{11} + a_{21}, \; a_{12} + a_{22} \, \right).$$

(a) Show that $\boldsymbol{T}$ is a linear map.

(b) Let $\mathcal{B} = \{\boldsymbol{E}_{11}, \boldsymbol{E}_{12}, \boldsymbol{E}_{21}, \boldsymbol{E}_{22}\}$ be the standard basis for $\mathcal{M}_{2\times 2}(\mathbb{R})$ and $\mathcal{A} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_4\}$ be the standard basis for $\mathbb{R}^4$. Write down the matrix representation of $\boldsymbol{T}$ with respect to $\mathcal{B}$ and $\mathcal{A}$.

QUESTION 1. (CONTINUED)

(c) Determine a basis for either null($\boldsymbol{T}$) OR range($\boldsymbol{T}$).

QUESTION 1. (CONTINUED)

(d) Determine the dimensions of null($\boldsymbol{T}$) AND range($\boldsymbol{T}$).

(e) Determine whether the following properties of $\boldsymbol{T}$ is true. You are to explain your answer.

(i) Is $\boldsymbol{T}$ injective?

(ii) Is $\boldsymbol{T}$ surjective?

QUESTION 2. **(10 Marks)**

Recall that $\mathcal{F}(\mathbb{R}, \mathbb{R})$ is the vector space comprising all functions with domain $\mathbb{R}$ and codomain $\mathbb{R}$.

(a) Consider the set $\mathcal{U} = \{ f \in \mathcal{P}(\mathbb{R}) \ : \ \deg(f) \geq 2023 \}$. Is $\mathcal{U}$ a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$?

QUESTION 2. (CONTINUED)

(b) Consider the set $\mathcal{V} = \{e^x f \in \mathcal{F}(\mathbb{R}, \mathbb{R}) \ : \ f \in \mathcal{P}(\mathbb{R})\}$. Is $\mathcal{V}$ a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$?

QUESTION 3. **(5 Marks)**

Let $\mathcal{V}$ be a vector space. Let S be a subset of $\mathcal{V}$ that is closed under vector addition and scalar multiplication. Show that S is a subspace of $\mathcal{V}$ if and only if S is nonempty.