NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER II EXAMINATION 2023–2024
MH1301 – DISCRETE MATHEMATICS

Apr/May 2024 TIME ALLOWED: 2 HOURS

Matriculation Number: | | | | | | | | | |

Seat Number: | | | | | |

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **ELEVEN (11)** pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. However, if you write your solutions on other pages, please indicate them clearly.
6. This examination paper is **NOT ALLOWED** to be removed from the examination hall.

For examiners only

Question	Marks
1 (20)	
2 (15)	
3 (15)	

Question	Marks
4 (15)	
5 (20)	
6 (15)	

TOTAL (100)

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QUESTION 1. **(20 Marks)**

(a) A toy manufacturer has 10 limited edition toy cars to distribute among four different stores: A, B, C, and D. How many ways are there to distribute these toy cars if each toy car is unique, and stores A and B must each receive 3 toy cars, while stores C and D must each receive 2 toy cars?

(b) The toy manufacturer also has 40 limited edition plush toys to distribute to stores A, B, C, and D. How many ways are there to distribute these plush toys if the toys are indistinguishable, and each store must receive at least 5 plush toys, with the additional condition that store A receives at most 10 plush toys?

Question 1 continues on page 3.

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(c) Give a combinatorial proof that for positive integers $m \leq n$,

$$\sum_{k=0}^{m-1} C(n+2-k, 2) = C(n+3, 3) - C(n+3-m, 3).$$

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End of Question 1.

QUESTION 2. **(15 Marks)**

(a) Prove that for any positive integer n, there exists two distinct positive integers a and b such that $n^a - n^b$ is divisible by 15.

(b) Prove that if 11 integers are selected from among $\{1, 2, \ldots, 20\}$, then the selection includes integers c and d such that $c - d = 2$.

End of Question 2.

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QUESTION 3. **(15 Marks)**

Let a_n be the number of ternary strings of length n that contain at least one 1 and at least one 2. Find a recurrence relation for a_n. You need not solve it.

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End of Question 3.

QUESTION 4. **(15 Marks)**

(a) Find all solutions of the recurrence relation

$$a_n = 3a_{n-1} - 2a_{n-2} + 2^n \qquad \text{for } n \geq 2.$$

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Question 4 continues on page 7.

(b) Find the solution of the recurrence relation in Part (a) with initial values $a_0 = 4$ and $a_1 = 11$.

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End of Question 4.

QUESTION 5. **(20 Marks)**

Determine whether each of the statements below is true or false. Give an explanation if it is true or provide a counterexample if it is false.

(a) Suppose G is a multigraph with 10 edges and every vertex has the same degree d. Then d must be $1, 2, 5$, or 10 and for each such d, at least one such graph exists.

(b) Suppose G is a simple graph consisting of two 3-cycles. (See diagram below.) By adding edges between the two 3-cycles, we can create a new graph that contains an Eulerian circuit.



Question 5 continues on page 9.

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(c) If T is a tree with more than two vertices, then it is bipartite.

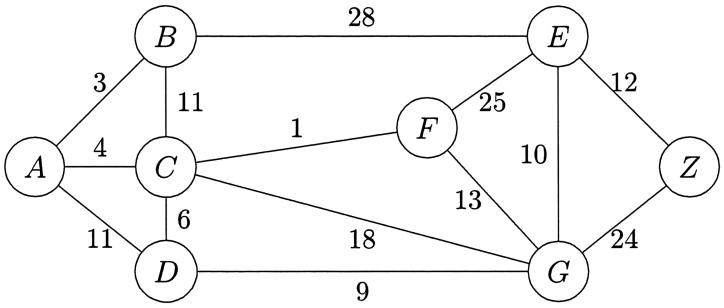
(d) There exists a 4-regular connected simple planar graph with 5 vertices.

End of Question 5.

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QUESTION 6. **(15 Marks)**

The following is an undirected weighted graph.



(a) Find the minimum spanning tree for the graph using Prim's algorithm starting with vertex A. List the order in which the edges are added, and compute the total weight of the tree.

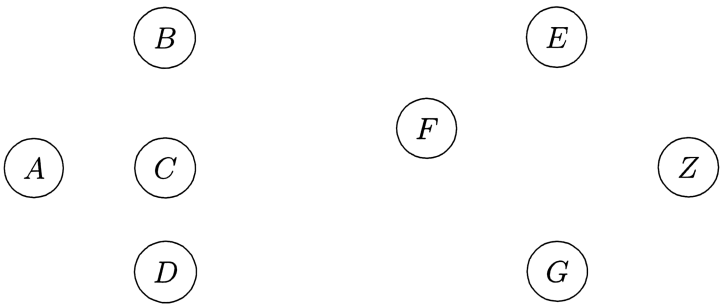
Question 6 continues on page 11.

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(b)(i) By using Dijkstra's algorithm to compute the shortest path between A and Z, fill in the table below illustrating how the values of $d[v]$ are updated.

$d[A]$	$d[B]$	$d[C]$	$d[D]$	$d[E]$	$d[F]$	$d[G]$	$d[Z]$	S
0	∞	∞	∞	∞	∞	∞	∞	$\emptyset$
0								
0								
0								
0								
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0								

(b)(ii) Complete the following spanning tree giving the minimum distances from vertex A to the other vertices.



END OF PAPER

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MH1301 DISCRETE MATHEMATICS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

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