MH1301 Discrete Mathematics

Tutorial 10

Academic Year 2025/2026, Semester 2

Quantitative Research Society @NTU

August 17, 2026

Overview

This tutorial focuses on planar graph bounds, bipartite planar graphs, regular planar graphs, and chromatic numbers of wheel graphs. The main tools are:

- Euler's formula for connected planar graphs;
- planar edge bounds for simple planar and bipartite planar graphs;
- degree counting via the Handshake Lemma;
- complement edge counting;
- wheel graphs and chromatic-number parity;
- bipartite graphs and odd-cycle obstructions;
- explicit planar constructions and non-planarity by density.

Question map.

- Question 1: Euler's formula, face degrees, and girth bounds.
- Question 2: planarity of $K_{2,n}$ and nonexistence of planar 6-regular simple graphs.
- Question 3: complement edge counting and unavoidable non-planarity.
- Question 4: planar cubic examples and planar edge upper bounds.
- Question 5: chromatic numbers of wheel graphs.
- Question 6: degree-1 graphs and 2-colourability.
- Question 7: adding edges to wheel graphs.
- Question 8: possible numbers of regions in connected planar graphs.
- Question 9: bipartite planar region bounds and a 6-vertex 8-region example.

For academic reference only. This document is provided for educational purposes and does not constitute an official question paper, answer key, or any formally authorised academic material. Its content is not guaranteed to be complete or accurate.

Contents

Question 1 (Euler formula and face-degree bounds) **4**
Problem . . . 4
Solution . . . 4
Method 1: Euler formula and direct face-degree counting . . . 4
Method 2: Planar girth inequality . . . 5

Question 2 (Planarity of $K_{2,n}$ and planar 6-regular graphs) **6**
Problem . . . 6
Solution . . . 6
Method 1: Explicit drawing and planar edge bound . . . 6
Method 2: General construction and average-degree obstruction . . . 7

Question 3 (A graph or its complement must be nonplanar) **8**
Problem . . . 8
Solution . . . 8
Method 1: Add the two planar edge bounds . . . 8
Method 2: Pigeonhole density argument . . . 9

Question 4 (Planar cubic examples and edge upper bounds) **10**
Problem . . . 10
Solution . . . 10
Method 1: K_4 example and Euler edge bound . . . 10
Method 2: Tetrahedral graph and maximal-planar obstruction . . . 11

Question 5 (Chromatic numbers of wheel graphs) **12**
Problem . . . 12
Solution . . . 12
Method 1: Wheel graph formula from rim parity . . . 12
Method 2: Lower bounds and explicit colourings . . . 13

Question 6 (Degree-one graphs and two-colourability) **14**
Problem . . . 14
Solution . . . 14
Method 1: Handshake Lemma and bipartite criterion . . . 14
Method 2: Matching construction and BFS colouring . . . 14

Question 7 (Adding edges to wheel graphs) **16**
Problem . . . 16
Solution . . . 16
Method 1: Completion to K_5 and planar edge bound . . . 16
Method 2: Kuratowski-minimal obstruction and density . . . 17

Question 8 (Connected planar graphs with prescribed number of regions) 18
Problem 18
Solution 18
Method 1: Euler formula and wheel construction 18
Method 2: Maximal-planar bound and triangulated wheel faces 18

Question 9 (Bipartite planar region bounds and a six-vertex example) 20
Problem 20
Solution 20
Method 1: Bipartite planar edge bound and octahedral construction 20
Method 2: Region-count contradiction and triangulated W_6 construction . . 21

Unofficial — QRS@NTU — qrsntu.org

Question 1 (Euler formula and face-degree bounds)

Problem

(a) Suppose that a connected planar graph has eight vertices, each of degree three. Into how many regions is the plane divided by a planar representation of this graph?

(b) Suppose that a connected planar simple graph with m edges and n vertices contains no simple circuits of length 4 or less. Show that

$$m \leq \frac{5}{3}n - \frac{10}{3}$$

if $n \geq 4$.

Solution

Method 1: Euler formula and direct face-degree counting

(a) **Theorem/criterion used.** For a connected planar graph with n vertices, m edges, and r regions,

$$n - m + r = 2.$$

Also, the Handshake Lemma gives

$$\sum_{v \in V} \deg(v) = 2m.$$

Here $n = 8$, and every vertex has degree 3. Hence

$$2m = 8 \cdot 3 = 24, \qquad m = 12.$$

By Euler's formula,

$$8 - 12 + r = 2.$$

Therefore

$$r = 6.$$

Thus

$$\boxed{6}$$

regions are formed.

(b) Since the graph has no simple circuit of length 4 or less, every face boundary has length at least 5. Let $\deg(R)$ denote the number of edge-sides on the boundary of a region R, with a bridge counted twice if it appears twice in a boundary walk. Then

$$\sum_{R} \deg(R) = 2m.$$

Unofficial QRS@NTU — qrs.ntu.org

Since every region has degree at least 5,

$$2m = \sum_R \deg(R) \geq 5r.$$

Hence

$$r \leq \frac{2m}{5}.$$

By Euler's formula,

$$r = m - n + 2.$$

Thus

$$m - n + 2 \leq \frac{2m}{5}.$$

Multiplying by 5 gives

$$5m - 5n + 10 \leq 2m.$$

Therefore

$$3m \leq 5n - 10,$$

and hence

$$\boxed{m \leq \frac{5}{3}n - \frac{10}{3}.}$$

Method 2: Planar girth inequality

Theorem/criterion used. If a connected simple planar graph has girth at least $g \geq 3$, then

$$m \leq \frac{g}{g-2}(n-2).$$

This follows from

$$2m = \sum_R \deg(R) \geq gr$$

and Euler's formula $r = m - n + 2$.

(a) The graph is 3-regular on 8 vertices. Hence

$$m = \frac{3 \cdot 8}{2} = 12.$$

Euler's formula gives

$$r = 2 - n + m = 2 - 8 + 12 = 6.$$

Therefore

$$\boxed{r = 6.}$$

(b) The hypothesis says that the graph has no cycle of length 3 or 4. Hence its girth is at least 5. Apply the planar girth inequality with $g = 5$:

$$m \leq \frac{5}{5-2}(n-2) = \frac{5}{3}(n-2) = \frac{5}{3}n - \frac{10}{3}.$$

Thus

$$\boxed{m \leq \frac{5}{3}n - \frac{10}{3}.}$$

Unofficial Q&A @ qrsntu.org

Question 2 (Planarity of $K_{2,n}$ and planar 6-regular graphs)

Problem

(a) Prove that $K_{2,5}$ is planar by drawing the graph without edges that cross each other. What about $K_{2,n}$?

(b) Prove that there are no 6-regular simple graphs that are planar. *(Hint: How many edges does a 6-regular graph have?)*

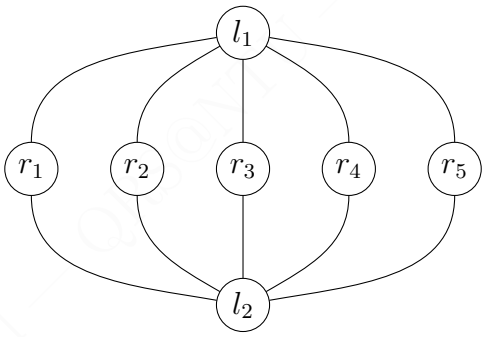
Solution

Method 1: Explicit drawing and planar edge bound

(a) The graph $K_{2,5}$ has bipartition

$$\{l_1, l_2\} \sqcup \{r_1, r_2, r_3, r_4, r_5\}.$$

A planar drawing is:



This drawing has no crossing edges.

More generally, $K_{2,n}$ is planar for every $n \geq 1$: place the n vertices $r_1, \ldots, r_n$ on a horizontal line, place l_1 above the line and l_2 below the line, draw all $l_1 r_i$ edges in the upper half-plane, and draw all $l_2 r_i$ edges in the lower half-plane. No two edges cross except at common endpoints. Hence

$$\boxed{K_{2,n} \text{ is planar for every } n \geq 1.}$$

(b) **Theorem/criterion used.** A simple planar graph with $n \geq 3$ vertices and m edges satisfies

$$m \leq 3n - 6.$$

If a graph is 6-regular on n vertices, then the Handshake Lemma gives

$$2m = \sum_{v \in V} \deg(v) = 6n.$$

Hence

$$m = 3n.$$

But a planar simple graph must satisfy

$$m \leq 3n - 6 < 3n.$$

This contradicts $m = 3n$. Therefore

$$\boxed{\text{there is no planar simple 6-regular graph.}}$$

Method 2: General construction and average-degree obstruction

(a) For $K_{2,n}$, write the bipartition as

$$L = \{u, v\}, \qquad R = \{x_1, \ldots, x_n\}.$$

Put $x_1, \ldots, x_n$ in order on a line, draw u above and v below. Draw all edges ux_i using pairwise noncrossing arcs above the line, and all edges vx_i using pairwise noncrossing arcs below the line. Since upper and lower arcs lie in disjoint half-planes, no crossing is produced. Hence

$$\boxed{K_{2,n} \text{ is planar.}}$$

(b) **Theorem/criterion used.** Every simple planar graph has average degree strictly less than 6. Indeed, from $m \leq 3n - 6$,

$$\frac{1}{n}\sum_{v \in V} \deg(v) = \frac{2m}{n} \leq \frac{2(3n-6)}{n} = 6 - \frac{12}{n} < 6.$$

A 6-regular graph has average degree exactly 6. This contradicts the strict planar average-degree bound. Therefore

$$\boxed{\text{no 6-regular simple graph is planar.}}$$

Question 3 (A graph or its complement must be non-planar)

Problem

Show that if G is a simple graph with at least 11 vertices, then either G or $\overline{G}$, the complement of G, is nonplanar.

Solution

Method 1: Add the two planar edge bounds

Theorem/criterion used. If a simple planar graph has $n \geq 3$ vertices and m edges, then

$$m \leq 3n - 6.$$

Also, for a simple graph G on n vertices,

$$|E(G)| + |E(\overline{G})| = \binom{n}{2}.$$

Suppose, for contradiction, that both G and $\overline{G}$ are planar. Let

$$m = |E(G)|.$$

Then

$$|E(\overline{G})| = \binom{n}{2} - m.$$

Since both graphs are planar,

$$m \leq 3n - 6$$

and

$$\binom{n}{2} - m \leq 3n - 6.$$

Adding,

$$\binom{n}{2} \leq 6n - 12.$$

Thus

$$\frac{n(n-1)}{2} \leq 6n - 12.$$

Multiplying by 2,

$$n(n-1) \leq 12n - 24,$$

so

$$n^2 - 13n + 24 \leq 0.$$

But for $n \geq 11$,

$$n^2 - 13n + 24 = n(n-13) + 24.$$

At $n = 11$ this equals

$$121 - 143 + 24 = 2 > 0,$$

and the quadratic is increasing for all $n \geq 11$. Hence the inequality is impossible for $n \geq 11$.

Therefore the assumption that both G and $\overline{G}$ are planar is false. Thus

$$\boxed{\text{for } n \geq 11, \text{ at least one of } G \text{ and } \overline{G} \text{ is nonplanar.}}$$

Method 2: Pigeonhole density argument

Theorem/criterion used. A simple planar graph on $n \geq 3$ vertices has at most $3n - 6$ edges.

Since

$$|E(G)| + |E(\overline{G})| = \binom{n}{2},$$

at least one of $G, \overline{G}$ has at least half of the total number of possible edges:

$$\max\{|E(G)|, |E(\overline{G})|\} \geq \frac{1}{2}\binom{n}{2} = \frac{n(n-1)}{4}.$$

For $n \geq 11$,

$$\frac{n(n-1)}{4} > 3n - 6.$$

Indeed,

$$\frac{n(n-1)}{4} > 3n - 6 \iff n^2 - n > 12n - 24 \iff n^2 - 13n + 24 > 0,$$

which holds for every integer $n \geq 11$.

Thus at least one of G and $\overline{G}$ has more than $3n-6$ edges. Such a graph cannot be planar. Hence

$$\boxed{\text{either } G \text{ or } \overline{G} \text{ is nonplanar.}}$$

Unofficial — QRS@NTU — qrs.ntu.org

Question 4 (Planar cubic examples and edge upper bounds)

Problem

(a) Find a planar 3-regular graph.

(b) Is there a planar graph with 10 vertices and 25 edges?

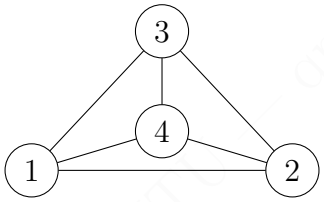
Solution

Method 1: K_4 example and Euler edge bound

(a) Take

$$G = K_4.$$

A planar drawing is:



Since every vertex of K_4 is adjacent to the other three vertices,

$$\deg(v) = 3 \qquad \text{for all } v \in V(K_4).$$

Hence K_4 is planar and 3-regular. Therefore

$$\boxed{K_4 \text{ is a planar 3-regular graph.}}$$

(b) **Theorem/criterion used.** A simple planar graph with $n \geq 3$ vertices and m edges satisfies

$$m \leq 3n - 6.$$

For $n = 10$,

$$m \leq 3(10) - 6 = 24.$$

A graph with 10 vertices and 25 edges would have

$$25 > 24,$$

contradicting the planar edge bound. Therefore

$$\boxed{\text{there is no planar simple graph with 10 vertices and 25 edges.}}$$

Unofficial QRS@NTU qrsntu.org

Method 2: Tetrahedral graph and maximal-planar obstruction

(a) The tetrahedral graph is the graph of a tetrahedron. Its graph is exactly K_4. It has four vertices and every vertex is incident to the three other vertices. Thus it is 3-regular. Since it is the edge graph of a convex polyhedron, it has a planar representation. Hence

$$\boxed{K_4 \text{ works.}}$$

(b) A maximal simple planar graph on $n \geq 3$ vertices triangulates the sphere and has exactly

$$3n - 6$$

edges. Therefore no simple planar graph on 10 vertices can have more than

$$3(10) - 6 = 24$$

edges. Since $25 > 24$,

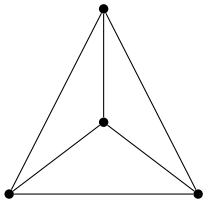
$$\boxed{\text{no such planar graph exists.}}$$

Unofficial — QRS@NTU — qrs.ntu.org

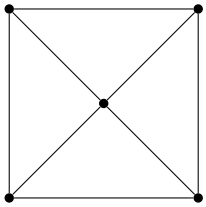
Question 5 (Chromatic numbers of wheel graphs)

Problem

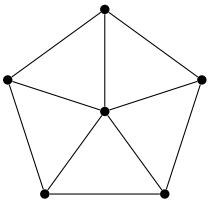
Determine the chromatic numbers of the following three graphs, called wheel graphs.



W_4



W_5



W_6

Solution

Method 1: Wheel graph formula from rim parity

Definition used. In this tutorial's notation, W_n is obtained by taking a cycle

$$C_{n-1}$$

and adding one central vertex adjacent to every vertex of the cycle.

The central vertex is adjacent to every rim vertex, so its colour cannot be used on the rim. Hence

$$\chi(W_n) = \chi(C_{n-1}) + 1.$$

For cycles,

$$\chi(C_k) = \begin{cases} 2, & k \text{ even}, \\ 3, & k \text{ odd}. \end{cases}$$

For W_4, the rim is C_3, so

$$\chi(W_4) = \chi(C_3) + 1 = 3 + 1 = 4.$$

For W_5, the rim is C_4, so

$$\chi(W_5) = \chi(C_4) + 1 = 2 + 1 = 3.$$

For W_6, the rim is C_5, so

$$\chi(W_6) = \chi(C_5) + 1 = 3 + 1 = 4.$$

Therefore

$$\boxed{\chi(W_4) = 4, \qquad \chi(W_5) = 3, \qquad \chi(W_6) = 4.}$$

Method 2: Lower bounds and explicit colourings

(a) For W_4, the graph is K_4: the rim is a triangle and the centre is adjacent to all three rim vertices. Since every pair of vertices is adjacent,

$$\chi(W_4) = 4.$$

Thus

$$\boxed{\chi(W_4) = 4.}$$

(b) For W_5, the rim is C_4. Colour the four rim vertices alternately with colours 1 and 2. The centre is adjacent to every rim vertex, so it must receive a third colour. Hence

$$\chi(W_5) \leq 3.$$

Also, W_5 contains a triangle formed by the centre and any rim edge, so

$$\chi(W_5) \geq 3.$$

Therefore

$$\boxed{\chi(W_5) = 3.}$$

(c) For W_6, the rim is C_5, an odd cycle. Thus the rim already requires three colours:

$$\chi(C_5) = 3.$$

The centre is adjacent to every rim vertex. In any proper colouring of C_5, all three colours appear on the rim. Hence the centre cannot use any of those three colours, so

$$\chi(W_6) \geq 4.$$

Four colours suffice by using three colours on the rim and a fourth colour on the centre. Hence

$$\boxed{\chi(W_6) = 4.}$$

Unofficial — QRS@NTU — qrs.ntu

Question 6 (Degree-one graphs and two-colourability)

Problem

(a) Under what condition on n is there a graph with n vertices where all vertices have degree 1?

(b) True or false: a planar graph that has no subgraph that is an odd cycle can be 2-colored.

Solution

Method 1: Handshake Lemma and bipartite criterion

(a) If all n vertices have degree 1, then

$$\sum_{v \in V} \deg(v) = n.$$

By the Handshake Lemma, the sum of all degrees must equal $2m$, so it must be even. Hence n must be even.

Conversely, if n is even and $n \geq 2$, pair the vertices:

$$\{v_1, v_2\}, \{v_3, v_4\}, \ldots, \{v_{n-1}, v_n\},$$

and add the edges

$$v_1v_2,\ v_3v_4,\ \ldots,\ v_{n-1}v_n.$$

This gives a disjoint union of $\frac{n}{2}$ copies of K_2, and every vertex has degree 1.

Therefore

$$\boxed{\text{such a graph exists exactly when } n \geq 2 \text{ is even.}}$$

(b) **Theorem/criterion used.** A graph is bipartite if and only if it has no odd cycle. A graph is 2-colourable if and only if it is bipartite.

The graph has no subgraph that is an odd cycle, so it has no odd cycle. Therefore it is bipartite, and hence it can be coloured with two colours. Planarity is not needed.

Thus the statement is

$$\boxed{\text{True.}}$$

Method 2: Matching construction and BFS colouring

(a) A graph in which every vertex has degree 1 is a perfect matching: every component is a single edge K_2. Hence the vertices are partitioned into pairs. Such a partition exists precisely when n is even. For ordinary positive vertex counts, this gives

$$\boxed{n \text{ even and } n \geq 2.}$$

(b) Assume the graph has no odd cycle. In each connected component, choose a root vertex and colour each vertex by the parity of its distance from the root:

$$\text{colour 0 if } \operatorname{dist}(r, v) \text{ is even}, \qquad \text{colour 1 if } \operatorname{dist}(r, v) \text{ is odd}.$$

If an edge joined two vertices of the same parity level, then the two shortest root paths together with that edge would produce an odd cycle. This is impossible. Hence every edge joins vertices of opposite colours.

Therefore the graph is 2-colourable, so the statement is

$$\boxed{\text{True.}}$$

Unofficial — QRS@NTU — qrsntu.org

Question 7 (Adding edges to wheel graphs)

Problem

(a) How many edges does one have to add to the wheel graph W_5 so that the resulting simple graph is non-planar?

(b) If we add three new edges to W_6 so that the result is a simple graph, can the result be planar?

Solution

Method 1: Completion to K_5 and planar edge bound

(a) The wheel graph W_5 has 5 vertices. It consists of a 4-cycle and a centre adjacent to all four rim vertices, so

$$|E(W_5)| = 4 + 4 = 8.$$

The complete graph K_5 has

$$\binom{5}{2} = 10$$

edges. Therefore W_5 is missing exactly

$$10 - 8 = 2$$

edges from K_5.

Adding these two missing edges produces K_5, which is nonplanar. Adding only one edge gives a 5-vertex graph with 9 edges. Since $K_5 - e$ is planar, one edge is not enough to force non-planarity from W_5; the resulting graph is a subgraph of $K_5 - e$ up to isomorphism and remains planar.

Therefore

$$\boxed{2}$$

edges are needed.

(b) The wheel graph W_6 has a 5-cycle as its rim and one central vertex. Hence

$$|E(W_6)| = 5 + 5 = 10.$$

Adding three new edges gives a simple graph with

$$m = 10 + 3 = 13$$

edges and

$$n = 6$$

vertices.

A simple planar graph with 6 vertices has at most

$$3n - 6 = 3(6) - 6 = 12$$

edges. Since

$$13 > 12,$$

the resulting graph cannot be planar. Hence

$$\boxed{\text{No.}}$$

Method 2: Kuratowski-minimal obstruction and density

(a) **Theorem/criterion used.** The smallest simple nonplanar graph on five vertices is K_5. Equivalently, every proper subgraph of K_5 is planar.

The graph W_5 has 5 vertices and is missing exactly the two diagonals of the outer 4-cycle. Adding both missing edges yields K_5, so two edges are sufficient. With only one added edge, the graph is still a proper subgraph of K_5, hence planar. Therefore

$$\boxed{\text{2 edges are necessary and sufficient.}}$$

(b) A planar simple graph on 6 vertices has at most 12 edges. The graph obtained from W_6 by adding three new simple edges has 13 edges. Hence it violates the planar edge bound and cannot be planar:

$$\boxed{\text{No, the result cannot be planar.}}$$

Unofficial — passed on as-is

Question 8 (Connected planar graphs with prescribed number of regions)

Problem

(a) Is there a connected planar graph with n vertices and $2n$ regions?

(b) Is there a connected planar graph with n vertices and n regions?

Solution

Method 1: Euler formula and wheel construction

(a) **Theorem/criterion used.** A simple planar graph with $n \geq 3$ vertices and m edges satisfies

$$m \leq 3n - 6.$$

For a connected planar graph,

$$r = m - n + 2.$$

Hence

$$r \leq (3n - 6) - n + 2 = 2n - 4.$$

Since

$$2n > 2n - 4,$$

a connected simple planar graph cannot have $2n$ regions. Therefore

$$\boxed{\text{No.}}$$

(b) Yes. For $n \geq 4$, take the wheel graph W_n. It has n vertices: one central vertex and a rim cycle on $n - 1$ vertices. Its number of edges is

$$m = (n - 1) + (n - 1) = 2n - 2.$$

Since W_n is connected and planar, Euler's formula gives

$$r = m - n + 2 = (2n - 2) - n + 2 = n.$$

Hence

$$\boxed{\text{Yes, } W_n \text{ is an example for } n \geq 4.}$$

Method 2: Maximal-planar bound and triangulated wheel faces

(a) In a simple connected planar graph, the maximum number of regions occurs when the graph is maximal planar. In that case,

$$m = 3n - 6$$

and hence

$$r = m - n + 2 = 3n - 6 - n + 2 = 2n - 4.$$

Thus no simple connected planar graph can reach $2n$ regions. Therefore

$$\boxed{\text{No connected simple planar graph has } 2n \text{ regions.}}$$

(b) The wheel graph W_n has a rim cycle of length $n - 1$ and one hub. Its planar drawing has:

- $n - 1$ triangular regions incident with the hub;
- one exterior region bounded by the rim cycle.

Therefore the total number of regions is

$$(n - 1) + 1 = n.$$

Hence

$$\boxed{\text{Yes, } W_n \text{ gives } n \text{ vertices and } n \text{ regions.}}$$

Unofficial — QRS@NTU — qrs.ntu.org

Question 9 (Bipartite planar region bounds and a six-vertex example)

Problem

(a) Let $n \geq 3$. Is there a connected, planar, bipartite graph with n regions and n vertices?

(b) Is there a connected planar graph with 6 vertices and 8 regions?

Solution

Method 1: Bipartite planar edge bound and octahedral construction

(a) **Theorem/criterion used.** A connected simple bipartite planar graph with $n \geq 3$ vertices and m edges satisfies

$$m \leq 2n - 4.$$

This follows because every face has boundary length at least 4, so

$$2m = \sum_{R} \deg(R) \geq 4r,$$

and Euler's formula gives the bound.

If such a graph had n vertices and n regions, then by Euler's formula,

$$n - m + n = 2.$$

Hence

$$m = 2n - 2.$$

But the bipartite planar edge bound gives

$$m \leq 2n - 4.$$

These two equations would imply

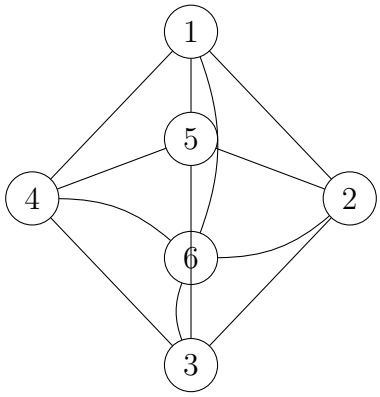
$$2n - 2 \leq 2n - 4,$$

impossible. Therefore

$$\boxed{\text{No such connected planar bipartite graph exists.}}$$

(b) Yes. Take the octahedral graph. It has 6 vertices and 12 edges. A planar drawing is:

Unofficial — QRS@NTU — qrs.ntu.edu.sg



By Euler's formula,

$$r = m - n + 2 = 12 - 6 + 2 = 8.$$

Hence there is a connected planar graph with 6 vertices and 8 regions:

$$\boxed{\text{Yes.}}$$

Method 2: Region-count contradiction and triangulated W_6 construction

(a) Since the graph is bipartite, it has no odd cycle. In particular, a planar embedding has no triangular face. Hence every face has length at least 4:

$$\deg(R) \geq 4 \qquad \text{for every region } R.$$

Thus

$$2m = \sum_R \deg(R) \geq 4r.$$

If $r = n$, then

$$2m \geq 4n, \qquad m \geq 2n.$$

But Euler's formula with $r = n$ gives

$$n - m + n = 2, \qquad m = 2n - 2.$$

Combining,

$$2n - 2 \geq 2n,$$

contradiction. Therefore

$$\boxed{\text{No.}}$$

(b) Start with W_6, which has 6 vertices and

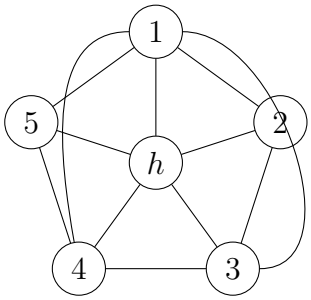
$$5 + 5 = 10$$

edges. Draw W_6 with the hub inside the rim pentagon. Add two noncrossing diagonals in the exterior face of the rim pentagon, for example from one rim vertex to two nonadjacent rim vertices. The result is still planar and simple, and it has

$$m = 10 + 2 = 12$$

edges.

A schematic drawing is:



By Euler's formula,

$$r = m - n + 2 = 12 - 6 + 2 = 8.$$

Hence

Yes, such a graph exists.

Unofficial — QRS@NTU — qrsntu.org