NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER I EXAMINATION 2025–2026

MH2100 – CALCULUS III

November 2025 TIME ALLOWED: 2 HOURS

SEAT NUMBER:

MATRICULATION NUMBER:

INSTRUCTIONS TO CANDIDATES

1. This question cum answer booklet contains **FIVE (5)** problems, each consisting of multiple parts, and comprises **TWENTY (20)** printed pages.
2. Attempt **ALL** problems. The number of marks allocated to each part of a problem is indicated after its statement.
3. Write your solution to each part of a problem in the **solution space provided**.
4. Write your matriculation number on the **cover page** and at the **bottom right corner of every odd numbered page**.
5. The last three pages are spill-over pages. Use them only if you run out of space for your solution(s), in which case clearly indicate the problem(s) that you are solving.
6. This question cum answer booklet is **not** to be removed from the exam venue.
7. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring in **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g., sticky notes, post-it notes, gluing or stapling of additional papers).
8. Candidates may use calculators, but they must systematically write down the steps of their work.
9. All work must be justified, unless the problem states otherwise.

PROBLEM 1.

(a) Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{x^8+y^8}{x^4+y^4}$. **(10 points)**

Problem 1 continues on the next page

(b) Determine whether or not the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$\forall x, y \in \mathbb{R}: \quad f(x,y) \stackrel{\text{df}}{=} \begin{cases} \dfrac{\sin(x^2+y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0); \\ 1, & \text{if } (x,y) = (0,0), \end{cases}$$

is totally differentiable at $(0,0)$. Justify your answer. **(10 points)**

Problem 1 continues on the next page Matric. No.:

End of Problem 1

PROBLEM 2.

A function $f : \mathbb{R}^3 \to \mathbb{R}$ is defined by

$$\forall x, y, z \in \mathbb{R} : \quad f(x, y, z) \stackrel{\text{df}}{=} x^2 + y^2 + z^2.$$

Let $C \stackrel{\text{df}}{=} \{(x, y, z) \in \mathbb{R}^3 \mid z^2 = x^2 + y^2 \text{ and } x - 2z = 3\}$.

(a) Show that C is a bounded and closed subset of $\mathbb{R}^3$. **(5 points)**

Problem 2 continues on the next page Matric. No.:

(b) Determine the absolute minimum and absolute maximum of $f|_C$. **(15 points)**

Note: You are required to apply the Method of Lagrange Multipliers.

Problem 2 continues on the next page

End of Problem 2

Matric. No.:

PROBLEM 3.

Let R denote the closed region in $\mathbb{R}_{>0} \times \mathbb{R}_{>0}$ that is bounded by the four curves

$$C_1 \stackrel{\text{df}}{=} \Big\{(x,y) \in \mathbb{R}_{>0} \times \mathbb{R}_{>0} \;\Big|\; \frac{y}{x} - x = 0\Big\},$$
$$C_2 \stackrel{\text{df}}{=} \Big\{(x,y) \in \mathbb{R}_{>0} \times \mathbb{R}_{>0} \;\Big|\; \frac{y}{x} - x = -2\Big\},$$
$$C_3 \stackrel{\text{df}}{=} \Big\{(x,y) \in \mathbb{R}_{>0} \times \mathbb{R}_{>0} \;\Big|\; \frac{y}{x} + x = 1\Big\},$$
$$C_4 \stackrel{\text{df}}{=} \Big\{(x,y) \in \mathbb{R}_{>0} \times \mathbb{R}_{>0} \;\Big|\; \frac{y}{x} + x = 2\Big\}.$$

Evaluate $\displaystyle\iint_R \frac{1}{x} \sin\Big(\frac{y}{x} + x\Big)\, \mathrm{d}A(x,y)$. **(20 points)**

Problem 3 continues on the next page

End of Problem 3

Matric. No.:

PROBLEM 4.

A plane vector field $\mathbf{F}$ on $\mathbb{R}^2$ is defined by

$$\forall x, y \in \mathbb{R}: \quad \mathbf{F}(x, y) \stackrel{\text{df}}{=} \big(ye^x, e^x + 3y^2\big).$$

(a) Show that $\mathbf{F}$ is conservative. **(5 points)**

(b) Derive a potential function f for $\mathbf{F}$. **(10 points)**

Problem 4 continues on the next page

(c) A vector function $\mathbf{r} : [0, 3] \to \mathbb{R}^2$ is defined by

$$\forall t \in [0,3]: \quad \mathbf{r}(t) \stackrel{\text{df}}{=} \begin{cases} \left(e^{t^2} - 1, e^{t^4} - 1\right), & \text{if } 0 \le t \le 2; \\ (e^{2t} - 1, e^{8t} - 1), & \text{if } 2 < t \le 3. \end{cases}$$

Let C denote the piecewise-smooth oriented curve in $\mathbb{R}^2$ parametrized by $\mathbf{r}$.
Evaluate $\displaystyle\int_C \mathbf{F} \bullet \mathrm{d}\mathbf{r}$. **(5 points)**

End of Problem 4 Matric. No.:

PROBLEM 5.

A space vector field $\mathbf{F}$ on $\mathbb{R}^3$ is defined by

$$\forall x, y, z \in \mathbb{R}: \quad \mathbf{F}(x, y, z) \stackrel{\text{df}}{=} \Big(xz - y^3 \cos(z), x^3 e^z, z e^{x^2+y^2+z^2} \Big).$$

Let S be a smooth oriented surface in $\mathbb{R}^3$, consisting of the upper hemisphere

$$\big\{ (x, y, z) \in \mathbb{R}^3 \;\big|\; x^2 + y^2 + z^2 = 1 \text{ and } z \geq 0 \big\}$$

equipped with the upward orientation.
Evaluate $\displaystyle\iint_S \operatorname{curl}(\mathbf{F}) \bullet \mathrm{d}\mathbf{S}$. **(20 points)**

Problem 5 continues on the next page

END OF PAPER

Matric. No.:

Spill-over page

If you require additional space for your solution(s) and plan to use this **spill-over page**, then please indicate as such in the designated solution space for the problem to refer the grader to here. Please ensure that the problem number and part are clearly labeled on this page.

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