MH2100 Calculus III

Midterm Practice 1 (Questions)

AY 2026/2027, Semester 1

Quantitative Research Society @NTU

Practice material

Overview

- **Coverage:** curves and tangents; limits, continuity, partial derivatives and total differentiability; chain rule, directional derivatives, tangent lines, linear approximation and the Implicit Function Theorem; extrema and Lagrange multipliers; order reversal, polar coordinates and volume by a height double integral.
- **Practice design:** four original problems, 100 marks in total; intended for 110 minutes. This practice design is not a confirmed 2026 examination format.
- **Current administration:** Tuesday 6 October 2026, 16:30–18:20, venue TBA. One double-sided typed A4 sheet is permitted; attachments and sticky notes are not. No calculator policy is stated here.

Unofficial practice material

Instructions

Answer **all** questions. Show all working clearly; answers without justification may receive zero marks.

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Question 1 (25 Marks)

PRACTICE PROBLEM 1.

(a) The curve C has parametrization $\mathbf{r}(t) = (t^2, t^3, 2t)$, $t \in \mathbb{R}$. Find the point on C at $t = 1$, a unit tangent vector there, and an equation of the tangent line. **(8 marks)**

(b) Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Determine whether f is continuous at $(0,0)$. Compute $f_x(0,0)$ and $f_y(0,0)$, and decide whether f is totally differentiable at $(0,0)$. Justify every conclusion. **(17 marks)**

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Question 2 (25 Marks)

PRACTICE PROBLEM 2.

(a) Let h be C^2 on a neighbourhood of $(1,2)$, with $h(1,2) = 3$ and $\nabla h(1,2) = (4,-1)$. Define $G(x,y) = h(x^2+y, xy+2)$. At $P = (1,0)$, find the directional derivative of G in the unit direction from P toward $(2,2)$, the tangent line to $G(x,y) = 3$, and the linear approximation of $G(1.02, -0.03)$. **(15 marks)**

(b) Let $F(x,y,z) = x^2 + ye^z + z^3 - 2$. Use the Implicit Function Theorem near $(1,0,1)$ to write $z = p(x,y)$. Find $p_x(1,0)$ and $p_y(1,0)$, then obtain the linear approximation of $p(1.04, -0.02)$. **(10 marks)**

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Question 3 (25 Marks)

PRACTICE PROBLEM 3. Let $f(x, y) = xy$ on the closed elliptical disk $C = \{(x, y) \in \mathbb{R}^2 : x^2 + 4y^2 \leq 4\}$.

(a) Find and classify every interior critical point of f. **(8 marks)**

(b) Use Lagrange multipliers to find every extremum candidate of f on $x^2 + 4y^2 = 4$. **(12 marks)**

(c) Establish that absolute extrema exist on C, then state the absolute maximum and minimum of f on C, including all points where they occur. **(5 marks)**

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Question 4 (25 Marks)

PRACTICE PROBLEM 4.

(a) Reverse the order of integration and evaluate

$$\int_0^1 \left[\int_{x^2}^{x} (1+2y)\,\mathrm{d}y \right] \mathrm{d}x.$$

(10 marks)

(b) Let $D = \{(x,y) \in \mathbb{R}^2 : x \geq 0,\ y \geq 0,\ 1 \leq x^2 + y^2 \leq 4\}$. Find the volume of the solid above D and below $z = 6 - x^2 - y^2$. Use polar coordinates and a height double integral. **(15 marks)**

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