MH2100 Calculus III

Midterm Practice 2 (Questions)

AY 2026/2027, Semester 1

Quantitative Research Society @NTU

Practice material

Overview

- **Coverage:** curves and tangent lines; limits, continuity, partial derivatives and total differentiability; chain rule, directional derivatives, tangent lines, linear approximation and the Implicit Function Theorem; extrema and Lagrange multipliers; order reversal, polar coordinates and volume by a height double integral.
- **Practice design:** four original problems, 100 marks in total; intended for 110 minutes. This practice design is not a confirmed 2026 examination format.
- **Current administration:** Tuesday 6 October 2026, 16:30–18:20, venue TBA. One double-sided typed A4 sheet is permitted; no attachments or sticky notes are permitted. No calculator policy is stated here.

Instructions

Answer **all** questions. Show all working clearly; answers without justification may receive zero marks.

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Question 1 (25 Marks)

PRACTICE PROBLEM 1.

(a) Let $\mathbf{r}(t) = (\mathrm{e}^t, \sin t, t^2)$. Find the point at $t = 0$, a unit tangent vector there, and an equation of the tangent line. **(8 marks)**

(b) Define $f : \mathbb{R}^2 \to \mathbb{R}$ by $f(0,0) = 0$ and $f(x,y) = x^2y/(x^2+y^2)$ otherwise. Determine continuity at the origin, compute both coordinate partial derivatives there, and decide total differentiability there. Justify each conclusion. **(17 marks)**

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Question 2 (25 Marks)

PRACTICE PROBLEM 2.

(a) Let h be C^2 near $(0,1)$, with $h(0,1)=2$, $\nabla h(0,1)=(3,2)$, and $h_{uu}(0,1)=2$, $h_{uv}(0,1)=1$, $h_{vv}(0,1)=-3$. Define $G(x,y)=h(x-y,xy)$. At $P=(1,1)$, find $G_{xy}(P)$, the directional derivative in the unit direction from P to $(3,0)$, the tangent line to $G=2$, and the linear approximation of $G(1.01,0.98)$. **(15 marks)**

(b) Let $F(x,y,z)=x+y^2+\mathrm{e}^z-2$. Use the IFT near $(0,1,0)$ to write $z=q(x,y)$. Find $q_x(0,1)$ and $q_y(0,1)$, then linearly approximate $q(0.03,0.99)$. **(10 marks)**

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Question 3 (25 Marks)

PRACTICE PROBLEM 3. Let $f(x, y) = x^2 - y^2$ on $C = \{(x, y) \in \mathbb{R}^2 : x^2 + 9y^2 \leq 9\}$.

(a) Find and classify every interior critical point. **(8 marks)**

(b) Use Lagrange multipliers to find every extremum candidate on $x^2+9y^2 = 9$. **(12 marks)**

(c) Use the Extreme Value Theorem to state all absolute extrema on C. **(5 marks)**

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Question 4 (25 Marks)

PRACTICE PROBLEM 4.

(a) Reverse the order of integration and evaluate

$$\int_0^1 \left[\int_{\sqrt{y}}^1 (1+x)\,\mathrm{d}x \right] \mathrm{d}y.$$

(10 marks)

(b) Let D be the part of $x^2 + y^2 \leq 4y$ in the first quadrant. Find the volume above D and below $z = 20 - x^2 - y^2$, using polar coordinates and a height double integral. **(15 marks)**

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