NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I

MH2100 – Calculus III

September 2018 TIME ALLOWED: 90 minutes

Name:

Matric. no.: Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. A two-sided A4 piece of paper with handwritten notes is permitted.

QUESTION 1. **(50 marks)**

Circle (only) the correct answer for each question. Each question is worth 5 marks.

(i) Find parametric equations to represent the line segment from $(-3, 4)$ to $(12, -8)$.

(a) $x = -3 - 15t, \quad y = 4 - 12t, \quad 0 \leq t \leq 1$

(b) $x = -3 + 15t, \quad y = 4 - 12t, \quad 0 \leq t \leq 2$

(c) $x = 8 - 15t, \quad y = 4 - 12t, \quad 0 \leq t \leq 2$

(d) $x = -3 + 15t, \quad y = 4 - 12t, \quad 0 \leq t \leq 1$

(e) $x = -3 - 15t, \quad y = 4 - 12t, \quad 0 \leq t \leq 2$

(ii) Find the point(s) on the following curve where the tangent is horizontal.

$$x = t^3 - 3t + 2, \quad y = t^3 - 3t^2 + 2$$

(a) $(0, 0)$

(b) $(0, 0)$ and $(2, -2)$

(c) $(2, 2)$ and $(-2, -2)$

(d) $(2, 2)$ and $(4, -2)$

(e) None of the above

(iii) Find parametric equations for the tangent line at the point when $t = 1$ to the curve with parametric equations

$$x = 3t, \quad y = 5t^2, \quad z = 4t^3.$$

(a) $x = 1 + 3t, \quad y = 1 + 10t, \quad z = 1 + 12t$

(b) $x = 3 + 3t, \quad y = 5 + 10t, \quad z = 4 + 12t$

(c) $x = 1 + 3t, \quad y = 1 + 5t, \quad z = 1 + 4t$

(d) $x = 3 + 3t, \quad y = 5 + 5t, \quad z = 4 + 4t$

(e) None of the above

Questions continue on the next page.

(iv) Find the unit tangent vector for the curve given by

$$\mathbf{r}(t) = \left\langle \frac{1}{5}t^5, \frac{1}{3}t^3, t \right\rangle.$$

(a) $\dfrac{\langle t^4, t^2, 1\rangle}{\sqrt{t^{10} + t^4}}$

(b) $\dfrac{\langle t^4, t^2, 1\rangle}{\sqrt{4t^8 + 4t^4 + 1}}$

(c) $\dfrac{\langle t^4, t^2, 1\rangle}{\sqrt{t^8 + t^4 + 1}}$

(d) $\dfrac{\langle t^4, t^2, 1\rangle}{\sqrt{t^8 + t^4}}$

(e) None of the above

(v) Find the directional derivative of the function $f(x, y, z) = xyz$ in the direction of the vector $\langle 5, -3, 2\rangle$.

(a) $\dfrac{1}{\sqrt{38}}(5yz - 3xz + 2xy)$

(b) $\dfrac{1}{\sqrt{38}}(5xy - 3xz + 2yz)$

(c) $5yz - 3xz + 2xy$

(d) $5xy - 3xz + 2yz$

(e) None of the above

(vi) Find the gradient vector of the function $f(x, y, z) = z^7 e^{2x\sqrt{y}}$.

(a) $e^{x^2\sqrt{y}}\langle 7xz\sqrt{y}, \dfrac{x^2 z}{2\sqrt{y}}, 2\rangle$

(b) $ze^{2x\sqrt{y}}\langle z\sqrt{y}, \dfrac{xz}{2\sqrt{y}}, 7z^5\rangle$

(c) $e^{x^2\sqrt{y}}\langle 2xz\sqrt{y}, \dfrac{x^2 z}{7\sqrt{y}}, 2\rangle$

(d) $ze^{2x\sqrt{y}}\langle 2z^6\sqrt{y}, \dfrac{xz^6}{\sqrt{y}}, 7z^5\rangle$

Questions continue on the next page.

(vii) Use the Chain Rule to find $\dfrac{dw}{dt}$ for

$$w = 3x^6y^3z, \quad x = 6t, \quad y = \cos 2t, \quad z = t\sin t.$$

(a) $3x^5y^2(36yz - 6xz\sin 2t + xy(\sin t + t\cos t))$

(b) $3x^6y^3(36yz - 6xz\sin 2t + xy(\sin t + t\cos t))$

(c) $648x^5y^2\sin 2t(\sin t + t\cos t)$

(d) $648x^6y^3\sin 2t(\sin t + t\cos t)$

(e) None of the above

(viii) Find an equation for the tangent plane to the surface given by the equation $xy + yz + xz = 38$ at the point $(4, 2, 5)$.

(a) $7x + 9y + 6z = 70$

(b) $7x + 9y + 6z = 74$

(c) $7x + 9y + 6z = 76$

(d) $7x + 9y + 6z = 78$

(e) None of the above

(ix) Find an equation for the tangent plane to the surface $z = 4x^3y^2 + 2y$ at the point $(1, -2, 12)$.

(a) $48x + 14y - z = 60$

(b) $48x - 14y - z = 64$

(c) $48x - 14y - z = 60$

(d) $48x + 14y - z = 64$

(e) None of the above

(x) Find and classify the critical points of the function $f(x, y) = e^{-2x}\sin 4y$ for $x \geq 0$ and $0 \leq y \leq \pi/2$.

(a) Local maximum $f(0, \pi/8) = 1$

(b) Saddle point $f(0, 0) = 0$

(c) Local minimum $f(0, \pi/4) = 0$

(d) None of the above

QUESTION 2. **(25 marks)**

(i) Evaluate each of the following limits or show that the limit does not exist. Justify your answers.

(a)
$$\lim_{(x,y)\to(6,6)} \frac{5e^{x-y}}{\cos(\pi(x-y))}$$

(b)
$$\lim_{(x,y)\to(0,0)} \frac{x^{21}y^{21}}{x^{42}+y^{84}}$$

(ii) Define
$$f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\frac{1}{x^2+y^2}\right); & \text{if } x^2+y^2 \neq 0 \\ 0; & \text{if } x^2+y^2 = 0. \end{cases}$$

(a) Is f continuous at $(0,0)$? Justify your answer.

(b) Find $f_x(0,0)$ and $f_y(0,0)$.

(c) Is f differentiable at $(0,0)$? Justify your answer.

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QUESTION 3. **(25 marks)**

Let

$$f(x, y) = y^3 - x^2 - 7y^2 + 2xy + 9y.$$

Find and classify all critical points of f. (For each critical point, determine whether or not it is a local max/min or a saddle point.)

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