NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I

MH2100 – Calculus III

September 2019 TIME ALLOWED: 90 minutes

Name:

Matric. no.: Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. **ONE** two-sided A4 piece of paper with handwritten notes is permitted.

QUESTION 1. **(30 marks)**

(i) Find parametric equations to represent the line segment from $(2, 3)$ to $(5, 5)$.

(ii) Write a vector equation for a parametrisation of the tangent line at the point when $t = \pi$ to the curve with parametric equations

$$x = \cos t, \quad y = \sin t, \quad z = t.$$

(iii) Write the parametrisation of a curve that has tangent vector $\langle e^t, 2t, \sin t \rangle$.

(iv) Find the point(s) on the following curve where the tangent is horizontal.

$$x = t + 4, \quad y = t^3 - 3t + 7$$

(v) Find the directional derivative of the function $f(x, y, z) = xy^2 - xz^2$ in the direction of the vector $\langle 1, 2, 3 \rangle$.

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QUESTION 2. **(35 marks)**

(i) Evaluate each of the following limits or show that the limit does not exist. Justify your answers.

(a)
$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y + 5}{\sin(xy) + 1}$$

(b)
$$\lim_{(x,y)\to(0,0)} \frac{7xy^3}{xy^3 - (x-y)^2}$$

(c)
$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2 + y^6}$$

(ii) Define
$$f(x,y) = \begin{cases} y\cos\left(\frac{xy}{x^2+y^2}\right); & \text{if } x^2+y^2 \neq 0 \\ 0; & \text{if } x^2+y^2 = 0. \end{cases}$$

(a) Is f continuous at $(0,0)$? Justify your answer.

(b) Find $f_x(0,0)$ and $f_y(0,0)$.

(c) Is f differentiable at $(0,0)$? Justify your answer.

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QUESTION 3. **(35 marks)**

(i) Find an equation for the tangent plane to the surface $z = 5x^2y - 7xy^2$ at the point $(3, 2, 6)$.

(ii) Find an equation for the tangent plane to the surface given by the equation $x^2y - yz + xz^2 = 17$ at the point $(1, 2, 5)$.

(iii) Use the Chain Rule to find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ for

$$f(x, y, z) = 2xy^2z^4, \quad x = 2uv, \quad y = u^2, \quad z = \sin v.$$

(iv) Find $\dfrac{\partial z}{\partial y}$ where z is defined implicitly by

$$xe^{4y} + 4y^2z^3 + ze^{8x} = 0.$$

(v) Let

$$f(x, y) = x^3 - 3x^2y + 3xy^2 - 3x - 9y.$$

Find and classify all critical points of f. (For each critical point, determine whether or not it is a local max/min or a saddle point.)

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