NANYANG TECHNOLOGICAL UNIVERSITY
MIDTERM EXAMINATION 2025–2026

MH2100 – CALCULUS III

October 2025 TIME ALLOWED: $1\frac{5}{6}$ HOURS

SEAT NUMBER:

MATRICULATION NUMBER:

INSTRUCTIONS TO CANDIDATES

1. This question-answer booklet contains **FOUR (4)** problems, each consisting of multiple parts, and comprises **FOURTEEN (14)** printed pages.
2. Attempt **ALL** problems. The number of marks allocated to each part of a problem is indicated after its statement.
3. Write your solution to each part of a problem in the **solution space provided**.
4. Write your matriculation number on the **cover page** and in the **bottom right corner of every odd numbered page**.
5. The last four pages are spill-over pages. Use them only if you run out of space for your solution(s), in which case clearly indicate the problem(s) that you are solving.
6. This question-answer booklet is **not** to be removed from Hall C.
7. This is a **restricted open-book** exam. You are allowed to consult only **ONE** double-sided help-sheet with typed or handwritten notes and **NO** attachments in the form of additional sheets, sticky notes, etc.
8. Candidates may use calculators, but they must systematically write down the steps of their work.
9. All work must be justified, unless the problem states otherwise.

PROBLEM 1.

(a) Evaluate $\lim_{(x,y)\to(2,1)} \dfrac{x^2y - x^2 - 4y + 4}{xy - x - 2y + 2}$. **(5 points)**

Problem 1 continues on the next page

(b) Determine whether or not the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by

$$\forall x, y \in \mathbb{R} : \quad f(x, y) \stackrel{\text{df}}{=} \sqrt[3]{x^3 + y^3},$$

is totally differentiable at $(0, 0)$. **(20 points)**

End of Problem 1

Matric. No.: _____

PROBLEM 2.

(a) A function $f : \mathbb{R}^3 \to \mathbb{R}$ is defined by

$$\forall x, y, z \in \mathbb{R} : \quad f(x, y, z) \stackrel{\text{df}}{=} z^2 - x^3 y^2.$$

Let $\mathbf{u}$ denote the unit vector pointing from $(1, -1, 1)$ toward $(2, 1, 0)$.
Compute $(D_{\mathbf{u}} f)(1, -1, 1)$ — with justification. **(10 points)**

Problem 2 continues on the next page

(b) Two functions, $f, g : \mathbb{R}^2 \to \mathbb{R}$, satisfy the following four properties:

- f is totally differentiable at $(1, 4)$.
- The tangent plane to the graph of f at $(1, 4, f(1, 4))$ is

$$\big\{(x, y, z) \in \mathbb{R}^3 \;\big|\; 3x + 5y = z + 26\big\}.$$

- $f_{\mathrm{xy}}(1, 4) = f_{\mathrm{yx}}(1, 4) = 1$ and $f_{\mathrm{xx}}(1, 4) = f_{\mathrm{yy}}(1, 4) = -1$.
- For all $x, y \in \mathbb{R}$, we have $g(x, y) = x \cdot f(x^2 - y, x + 2y)$.

Compute $g_{\mathrm{xy}}(-2, 3)$ — with justification. **(15 points)**

End of Problem 2

Matric. No.: _____

PROBLEM 3.

A function $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by

$$\forall x, y \in \mathbb{R} : \quad f(x, y) \stackrel{\text{df}}{=} e^{-xy}.$$

Let $C \stackrel{\text{df}}{=} \{(x, y) \in \mathbb{R}^2 \mid x^2 + 4y^2 \leq 1\}$.

(a) Determine all the critical point(s) of $f|_C$. **(5 points)**

Note: Every critical point of $f|_C$ belongs — by definition — to the interior of C.

(b) Determine the nature of each critical point of $f|_C$. **(5 points)**

Note: Apply the Second-Derivative Test.

Problem 3 continues on the next page

(c) Solve the Lagrange-multiplier problem for minimizing/maximizing f on

$$\mathrm{Bd}(C) = \big\{(x, y) \in \mathbb{R}^2 \;\big|\; x^2 + 4y^2 = 1\big\},$$

which is the boundary of C in $\mathbb{R}^2$. **(10 points)**

Note: You are required to apply the Method of Lagrange Multipliers.

Problem 3 continues on the next page Matric. No.: _____

(d) Explain why $f|_C$ attains an absolute minimum/maximum. **(3 points)**

(e) Determine the absolute minimum and absolute maximum of $f|_C$. **(2 points)**

End of Problem 3

PROBLEM 4.

(a) Evaluate $\displaystyle\int_0^1 \left[\int_0^{\sqrt{\frac{1-x}{3}}} e^{y-y^3} \, \mathrm{d}y \right] \mathrm{d}x.$ **(10 points)**

Hint: Perform a change of order of integration.

Problem 4 continues on the next page Matric. No.: _____

(b) Evaluate $\displaystyle\int_0^2 \left[\int_0^{\sqrt{2y-y^2}} \sqrt{x^2+y^2}\, \mathrm{d}x \right] \mathrm{d}y.$ **(15 points)**

Hint: Perform a transformation from Cartesian to polar coordinates.

END OF PAPER

Spill-over page

If you require additional space for your solution(s) and plan to use this **spill-over page**, then please indicate as such in the designated solution space for the problem to refer the grader to here. Please ensure that the problem number and part are clearly labeled on this page.

Matric. No.: _____

Spill-over page

If you require additional space for your solution(s) and plan to use this **spill-over page**, then please indicate as such in the designated solution space for the problem to refer the grader to here. Please ensure that the problem number and part are clearly labeled on this page.

Spill-over page

If you require additional space for your solution(s) and plan to use this **spill-over page**, then please indicate as such in the designated solution space for the problem to refer the grader to here. Please ensure that the problem number and part are clearly labeled on this page.

Matric. No.: _____

Spill-over page

If you require additional space for your solution(s) and plan to use this **spill-over page**, then please indicate as such in the designated solution space for the problem to refer the grader to here. Please ensure that the problem number and part are clearly labeled on this page.