MH3100 Real Analysis I

Midterm Practice 2 (Questions)

AY 2026/2027, Semester 1 – QRS original

Quantitative Research Society @NTU

October 12, 2026

TIME ALLOWED: 60 minutes.

Instructions to Students

1. This test contains **FOUR (4)** questions.
2. Answer **all** questions. The marks for each question are indicated below.
3. Write your answers in the box provided. Use the **facing page** for more space.
4. This is a **CLOSED BOOK** test.
5. You may use calculators. You may cite any results proved in the lectures and tutorials, **unless you are asked by the question to prove the result.**

Question	1	2	3	4	Total
Total Marks	6	6	6	6	24
Awarded Marks					

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Question 1 (6 Marks)

Is each of the following statement true or false? You do NOT need to justify your answer.

(a) (2 marks) If $\{a_n\}$ is a bounded sequence such that every convergent subsequence of $\{a_n\}$ converges to the same real number L, then $\{a_n\}$ converges to L.

(b) (2 marks) There is a monotone sequence that has a convergent subsequence but is itself divergent.

(c) (2 marks) If $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences with $b_n \neq 0$ for every $n \in \mathbb{N}$, then $\left\{\frac{a_n}{b_n}\right\}$ is a Cauchy sequence.

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Question 2 (6 Marks)

Is each of the following statement true or false? You do NOT need to justify your answer.

(a) (2 marks) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

(b) (2 marks) Every finite subset of $\mathbb{R}$ is a closed set.

(c) (2 marks) $\overline{\mathbb{Q} \cap [0,1]} = [0,1]$.

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Question 3 (6 Marks)

In this question, A is a nonempty set of real numbers that is bounded below, and $c > 0$. Write down the definition of $s = \inf A$, and use the definition to show that

$$\inf(cA) = c \inf A, \qquad \text{where } cA := \{ca : a \in A\}.$$

Do not use the ε-Criterion for Suprema and Infima.

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Continue your answer to Question 3 here:

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Question 4 (6 Marks)

In this question, $\{a_n\}$ is a sequence of nonnegative real numbers. Write down the definition of $a = \lim a_n$, and use the definition to show that if $a = \lim a_n$, then $\sqrt{a} = \lim \sqrt{a_n}$.

Do not use the Algebraic Limit Theorem.

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Continue your answer to Question 4 here:

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Answer to Question continues here: (Use this page if you need more space.)

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