MH3100 Real Analysis I

Midterm Practice 4 (from past Finals) (Questions)

Past Final questions in Midterm scope (Lectures 1–17)

Quantitative Research Society @NTU

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About This Practice Paper

This is QRS@NTU's fourth MH3100 Midterm practice paper. Instead of a fresh QRS-authored paper, it collects every question part from NTU's three most recent MH3100 Real Analysis I *Final* examinations (AY2022/23, AY2023/24, AY2024/25, Semester 2) that falls inside the **Midterm scope, Lectures 1–17** (construction and completeness of $\mathbb{R}$ through sets of real numbers). Parts that require Lecture 18–20 material (function limits, continuity, uniform continuity), differentiation, integration, uniform convergence of function sequences/series, or open covers/Heine-Borel are outside the Midterm syllabus and are omitted below; the scope table gives the reason for every omission. Since this paper draws only on genuine past-Final material, its total (97 marks across 8 questions) does not match the 24-mark, 60-minute Midterm format – treat it as extra practice, not a timed mock.

Solutions in this paper are held to the same three writing expectations as the Midterm: complete sentences; related statements joined by logical connectives; and inference explicitly marked with "therefore/hence/so".

Overview

Papers used (all MH3100 Real Analysis I, NTU, CLOSED BOOK, 6 questions, 2 hours):

- Semester 2 Examination 2022–2023 (April 2023).
- Semester II Examination 2023–2024 (April 2024).
- Semester II Examination 2024–2025 (April 2025).

In every one of the three papers, Question 1 (15 marks, five True/False parts) and Question 2 (15 marks, three existential parts) mix in-scope and out-of-scope sub-parts, Question 3 (20 marks) is a supremum/infimum proof that is fully in scope, and Questions 4–6 (20+20+10 marks) all hinge on continuity and/or uniform convergence and are fully out of scope. Marks are not printed per sub-part on the original papers for Questions 1–2; QRS distributes each question's total evenly across its parts.

Scope table

AY2022/23 Semester 2 Final

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Question	Status	Reason
Q1(a)–(c)	Partly (3/5)	Monotone Convergence Lemma (L9); Theorem on Subsequential Limits (L7); limit points (L13) – all in scope.
Q1(d)–(e)	Out	Uniform continuity; power series and uniform convergence – beyond L17.
Q2(a)	Partly (1/3)	Absolute convergence + bounded sequence, via Comparison/Absolute Convergence Test (L12).
Q2(b)–(c)	Out	Function limits (L18–20 topic); pointwise convergence of continuous functions – beyond L17.
Q3	In	Definition of supremum (L2), fully in scope.
Q4	Out	Hypothesis "f continuous" is L18–20 material.
Q5	Out	Uniform convergence of a function series – not covered in L1–17.
Q6	Out	Uniform convergence of continuous functions – not covered in L1–17.

AY2023/24 Semester 2 Final

Question	Status	Reason
Q1(a)–(c)	Partly (3/5)	Suprema of sets (L2); Bolzano-Weierstrass Theorem (L8); isolated points/closed sets (L13, L15) – all in scope.
Q1(d)–(e)	Out	Differentiability; uniform convergence of derivatives – beyond L17.
Q2(a)	Partly (1/3)	Series with $\sum a_n$ convergent but $\sum a_n^2$ not (L10–12).
Q2(b)–(c)	Out	Continuity and uniform continuity – beyond L17.
Q3	In	Definition of supremum (L2), fully in scope.
Q4	Out	Continuous functions f, g – L18–20 material.
Q5	Out	Uniform convergence of a function series.
Q6	Out	Function limits (L18–20 topic).

AY2024/25 Semester 2 Final

Question	Status	Reason
Q1(a)–(c)	Partly (3/5)	Sup/inf (L2); Cauchy Criterion for Series (L10) with the Archimedean Property (L3); closure/closed sets (L14–15) – all in scope.
Q1(d)–(e)	Out	Function limits (L18–20 topic); Weierstrass-M-test-style uniform convergence.
Q2	Out (0/3)	(a)–(b) continuous-function hypotheses; (c) uniform convergence – all beyond L17.
Q3	In	Sup/inf via rational upper bounds (L2) + Archimedean Property (L3), fully in scope.
Q4	Out	Continuous functions and compactness combined; the continuity hypothesis is L18–20 material (compactness itself, L16, would be in scope).
Q5	Out	Uniform convergence of a function series.
Q6	Out	Continuity + uniform convergence.

Total marks in this practice paper

Question	1	2	3	4	5	6	7	8	Total
Marks	9	5	20	9	5	20	9	20	97

Question 1 (9 Marks)

For each of the following statements, determine if it is true or false. You do not need to justify your answer.

(a) A monotone sequence is convergent if and only if it has a convergent subsequence.

(b) A sequence is divergent if it has a divergent subsequence.

(c) If a set has no limit points, then it is finite.

(Source: MH3100 Final AY22-23 Sem 2, Question 1(a)–(c). Question 1(d)–(e), on uniformly continuous functions and uniform convergence of power series, fall outside the Lecture 1–17 Midterm scope and are omitted; see the scope table above. The Final prints only the question total, 15 marks, for all five parts; QRS distributes this evenly, 3 marks per part.)

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Question 2 (5 Marks)

For each of the following existential statements, either give an example fulfilling the statement or prove that it is not true.

There is an absolutely convergent series $\sum_{n=1}^{\infty} a_n$ and bounded sequence $\{b_n\}_{n=1}^{\infty}$ for which the series $\sum_{n=1}^{\infty} a_n b_n$ is divergent.

(Source: MH3100 Final AY22-23 Sem 2, Question 2(a). Question 2(b)–(c), on function limits and pointwise convergence of continuous functions, fall outside the Lecture 1–17 Midterm scope and are omitted; see the scope table above. The Final's Question 2 total, 15 marks, is distributed evenly, 5 marks per part.)

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Question 3 (20 Marks)

Use the definition of supremum to prove that if two nonempty sets $A, B \subseteq \mathbb{R}$ that are bounded above satisfy $\sup A < \sup B$, then for all $a \in A$, there exists $b \in B$ such that $a < b$. Give an example to show that the converse is not true.

You do not need to show that the suprema exist.

(Source: MH3100 Final AY22-23 Sem 2, Question 3, verbatim, 20 marks.)

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Question 4 (9 Marks)

For each of the following statements, determine if it is true or false. You do not need to justify your answer.

(a) If the set A has supremum s and the set B has supremum t, then their intersection $A \cap B$ has supremum $\min\{s, t\}$.

(b) If an unbounded sequence has a bounded subsequence, then it has a convergent subsequence.

(c) If a set contains only isolated points, then it is closed.

(Source: MH3100 Final AY23-24 Sem 2, Question 1(a)–(c). Question 1(d)–(e), on differentiability and uniform convergence of derivatives, fall outside the Lecture 1–17 Midterm scope and are omitted; see the scope table above. Marks are again distributed evenly, 3 marks per part.)

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Question 5 (5 Marks)

For each of the following existential statements, either give an example fulfilling the statement or prove that it is not true.

There is a convergent series $\sum_{n=1}^{\infty} a_n$ such that the series $\sum_{n=1}^{\infty} a_n^2$ is not convergent.

(Source: MH3100 Final AY23-24 Sem 2, Question 2(a). Question 2(b)–(c), on continuous functions and Cauchy sequences and on uniform continuity, fall outside the Lecture 1–17 Midterm scope and are omitted; see the scope table above. Marks distributed evenly, 5 marks per part.)

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Question 6 (20 Marks)

Use the definition of supremum to prove that if a set $A \subseteq \mathbb{R}$ is nonempty and bounded above and B denotes the set $\{b \in \mathbb{R} : \exists a \in A,\ b < a\}$, then B has a supremum $\sup B$ and the supremum satisfies $\sup B = \sup A$.

(Source: MH3100 Final AY23-24 Sem 2, Question 3, verbatim, 20 marks. Unlike Question 3 of the other two Finals in this paper, this question does not waive the requirement to show the supremum exists.)

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Question 7 (9 Marks)

For each of the following statements, determine if it is true or false. You do not need to justify your answer.

(a) Two sets $A, B \subseteq \mathbb{R}$ that satisfy $(\forall a \in A,\, \forall b \in B,\, a < b)$ must satisfy $\sup A < \inf B$.

(b) A series $\sum_{n=1}^{\infty} a_n$ that satisfies $(m > n \implies |a_{n+1} + a_{n+2} + \cdots + a_m| < \frac{1}{n})$ for all natural numbers $m, n \in \mathbb{N}$ is convergent.

(c) A set is closed if and only if it is the closure of some set.

(Source: MH3100 Final AY24-25 Sem 2, Question 1(a)–(c). Question 1(d)–(e), on function limits and uniform convergence of a series of functions, fall outside the Lecture 1–17 Midterm scope and are omitted; see the scope table above. Marks distributed evenly, 3 marks per part.)

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Question 8 (20 Marks)

Use the definitions of suprema and infima to prove that if a set $A \subseteq \mathbb{R}$ and its set $B := \{b \in \mathbb{Q} : \forall a \in A,\ a \leq b\}$ of all rational upper bounds are nonempty, then $\sup A = \inf B$.

You do not need to show that the supremum and infimum exist.

(Source: MH3100 Final AY24-25 Sem 2, Question 3, verbatim, 20 marks.)

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