# NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2024–2025

MH3701 – BASIC OPTIMIZATION

April 2025 Time Allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **FOUR (4)** printed pages.
2. Answer each question beginning on a **FRESH** page of the answer book.
3. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers)
4. Calculators may be used. However, you should write down systematically the steps in the workings.

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## QUESTION 1 **(30 marks)**

(a) Consider the following tableau for an *artificial problem* of a linear program (P). Apply the simplex method, with the largest coefficient rule for entering variables

| z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $w_1$ | $w_2$ | RHS |
|---|---|---|---|---|---|---|---|
| 1 | 9 | 1 | 11 | 0 | 0 | 0 | 18 |
| 0 | -9 | 3 | 6 | 1 | 0 | 0 | 6 |
| 0 | 4 | 4 | 6 | 0 | 1 | 0 | 8 |
| 0 | 5 | -3 | 5 | 0 | 0 | 1 | 10 |

and the smallest subscript rule for leaving variables, on the artificial problem to either

- conclude that the linear program (P) is infeasible; or
- determine a feasible tableau for the linear program (P). You do not need to specify the z-row.

(b) Consider the following tableau

| z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | RHS |
|---|---|---|---|---|---|---|---|---|
| 1 | $a$ | $b$ | $c$ | $d$ | 0 | 0 | 0 | $e$ |
| 0 | 5 | 1 | -5 | 2 | 1 | 0 | 0 | 3 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 2 |
| 0 | -7 | 2 | -1 | -2 | 0 | 0 | 1 | 4 |

(i) Suppose that this is a tableau for a maximization linear program. Determine the conditions on $a, b, c, d$ that ensure the current tableau is optimal.

(ii) Suppose that $a = 1, b = 2, c = -2, d = 5$, can you conclude that this linear program is unbounded? Justify your answer.

(iii) Suppose that the column for $x_6$ represents an artificial variable $w_1$ and the column for $x_7$ represents an artificial variable $w_2$. Determine the values of $a, b, c, d$ and $e$ to construct a tableau for the artificial problem that includes only these two artificial variables, $w_1$ and $w_2$.



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## QUESTION 2 (25 marks)

Consider the following program:

$$
\begin{array}{lrcrcrcrcrcrcl}
\text{maximize} & c_1x_1 & + & c_2x_2 & + & c_3x_3 & + & c_4x_4 & + & c_5x_5 & + & c_6x_6 & & \\
\text{subject to} & -x_1 & & & + & x_3 & + & x_4 & + & x_5 & - & 2x_6 & = & 4 \\
& 4x_1 & + & 2x_2 & + & x_3 & - & 6x_4 & + & 8x_5 & & & = & 2 \\
& -2x_1 & - & x_2 & + & x_3 & + & 3x_4 & - & x_5 & - & 3x_6 & = & 5 \\
& & & & & x_1, x_2, x_3, x_4, x_5, x_6 & & & \geq & 0 & & & &
\end{array}
$$

Suppose we are given a feasible basis $\{x_1, x_3, x_4\}$.

(a) If the objective coefficient $(c_1, c_2, c_3, c_4, c_5, c_6) = (-5, -2, -1, 5, -5, 2)$, starting from the given basis and apply the simplex method in *algebraic form* to solve the linear program. If the linear program is unbounded, give a sequence of feasible solutions to demonstrate its unboundedness.

(b) Determine the conditions on the objective coefficients $c_1, c_2, c_3, c_4, c_5$, and $c_6$ such that the linear program has at least an optimal solution and $\{x_1, x_3, x_4\}$ is an optimal basis.

## QUESTION 3 (25 marks)

Consider a linear program:

$$
\begin{array}{lrcrcrcl}
\text{maximize} & -x_1 & + & 2x_2 & + & x_3 & & \\
\text{subject to} & -2x_1 & & & + & x_3 & \leq & 1 \\
& 4x_1 & - & 2x_2 & + & x_3 & \leq & 4 \\
& -x_1 & + & x_2 & + & 2x_3 & = & 2 \\
& & & & & x_2, x_3 & \geq & 0
\end{array}
$$

(a) Write down its dual linear program. Use $y_1$, $y_2$, $y_3$ to denote the dual variables for the primal constraints.

(b) Write down the complementary slackness conditions.

(c) Use the Complementary Slackness Theorem to determine if the given solution $(x_1, x_2, x_3) = (4, 6, 0)$ is optimal.



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(d) Write down the artificial problem for this linear program, using $s_i$(s) to denote the slack variable(s) and $w_i$(s) to denote the artificial variable(s).

**QUESTION 4** **(20 marks)**

Determine if each of the following statements is true or false. Please justify your answer.

(a) A linear program is unbounded if and only if its feasible region is unbounded.

(b) If there does not exist a vector $x$ of nonnegative real numbers satisfying the linear equation $Ax = b$, where $A$ is a matrix with $m$ rows and $n$ columns and $b$ is a vector with $m$ entries, then there must exist a vector of $y$ with $m$ entries such that each entry of $A^T y$ is nonnegative while $b^T y$ is negative.

**END OF PAPER**

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

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