NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER II EXAMINATION 2024-2025

MH4110 – Partial Differential Equations

April 2025 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **RESTRICTED OPEN BOOK** exam. You are only allowed to bring into the examination hall **ONE DOUBLE-SIDED A4-SIZE REFERENCE SHEET WITH TEXTS HANDWRITTEN OR TYPED ON THE A4 PAPER WITHOUT ANY ATTACHMENTS** (e.g. sticky notes, post-it notes, gluing or stapling of additional papers).
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

QUESTION 1 (15 marks)

Given the function $u(x,t) = A\cos(kx - \omega t + \delta)$, where A, k, and ω are positive constants, and δ is a real number, identify a first-order partial differential equation that this function satisfies. Additionally, determine the characteristic curves associated with this first-order partial differential equation.

QUESTION 2 (15 marks)

Find the solution of the first-order partial differential equation for $u = u(x,y)$

$$u_x - xu_y + 3u = 1.$$

QUESTION 3 (15 marks)

Find the solution $u = u(x,y)$ of the partial differential equation

$$4u_{xx} - 4u_{xy} + u_{yy} = xy.$$

QUESTION 4 (30 marks)

Let u be the solution to the initial-boundary value problem for the wave equation:

$$\begin{cases} u_{tt} - 4u_{xx} + u_t = 0, & 0 < x < 1,\ t > 0, \\ u(0,t) = u(1,t) = 0, & t \geq 0, \\ u(x,0) = 0, \quad u_t(x,0) = x(1-x), & 0 \leq x \leq 1. \end{cases}$$

(a) Prove that the solution $u(x,t)$ to this problem is unique in the domain $(x,\ t) \in (0,\ 1) \times (0,\ +\infty)$.

(b) Find the solution $u(x,t)$.

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

QUESTION 5 (15 marks)

Find the solution of the inhomogeneous problem,

$$\begin{cases} u_t - u_{xx} = e^{-x^2}\delta(t-2), & -\infty < x < \infty, \ \ t > 0, \\ u(x,0) = 0, & -\infty < x < \infty. \end{cases}$$

QUESTION 6 (10 marks)

Please write down the partial differential equation that your group presented for the group project during the semester, specifically Maxwell's equations or the elastic wave equation. Additionally, provide a general overview of how the equation you presented is derived, ensuring the explanation does not exceed 200 words.

END OF PAPER

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library

MH4110 PARTIAL DIFFERENTIAL EQUATIONS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

ATTENTION: The Singapore Copyright Act applies to the use of this document. Nanyang Technological University Library